Separation Problems in Logic

Jędrzej Kołodziejski

6 XI 2025
Warszawa

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(with a slight algorithmic bias)

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Separators

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Consider **objects** of some sort

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(natural numbers, graphs, programs...)

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Consider **objects** of some sort

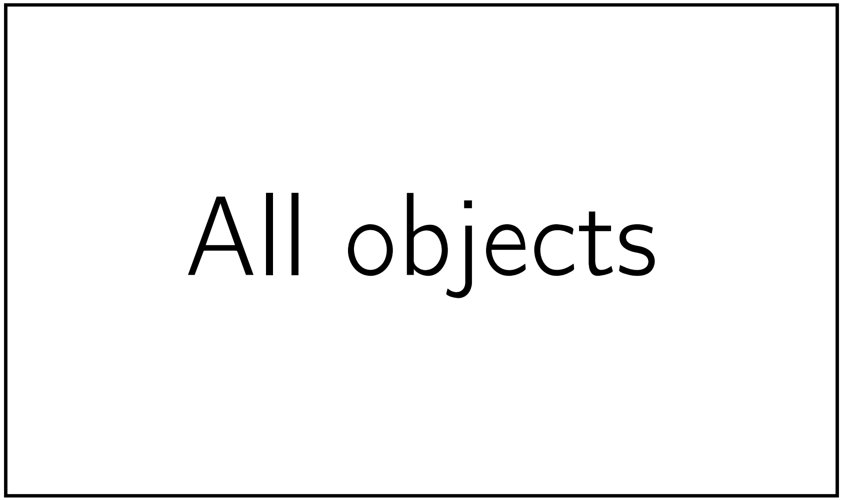
(natural numbers, graphs, programs...)

and some **properties** of these objects

Separators

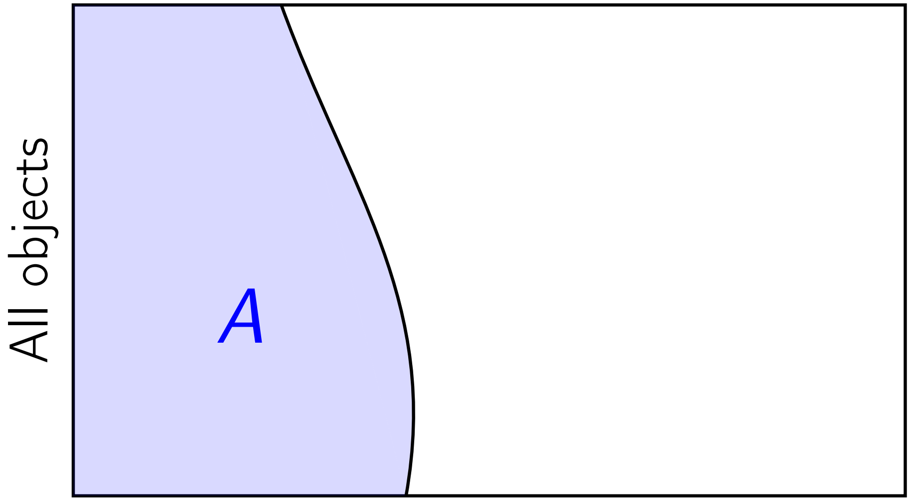
All objects

Separators



given properties A, B
which exclude each other:

Separators



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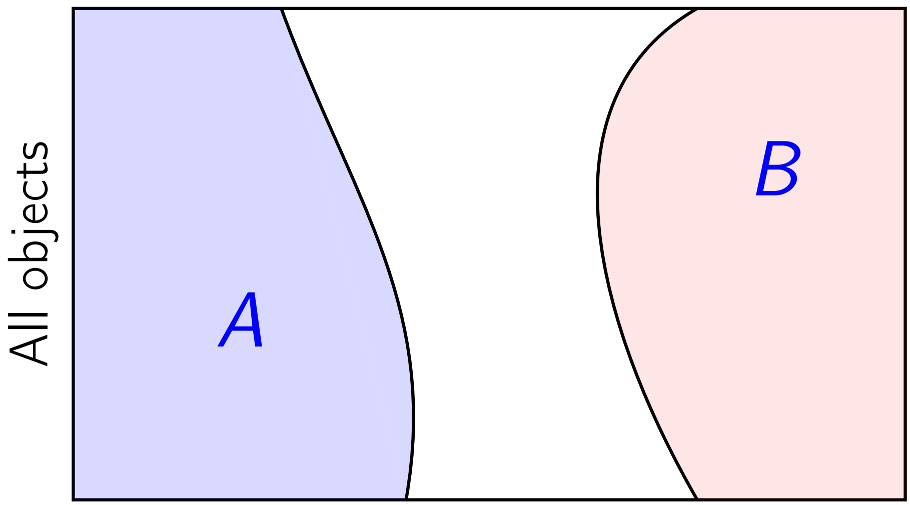

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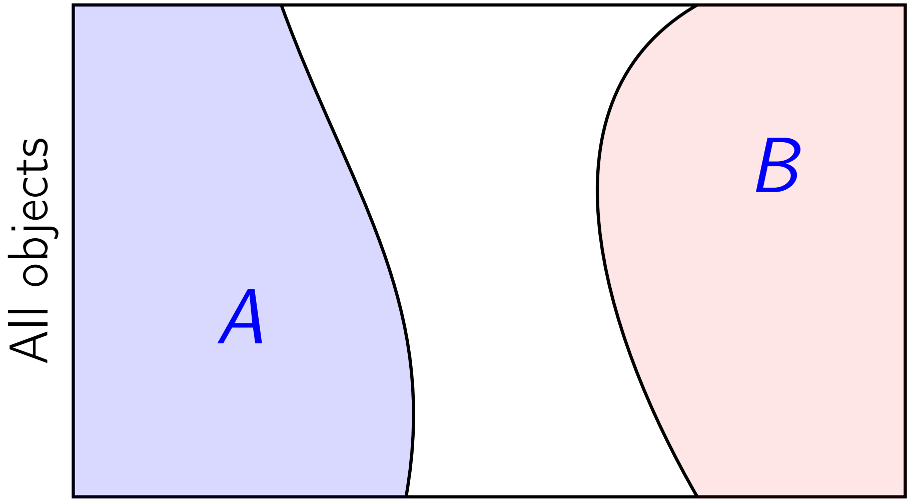

given properties A, B
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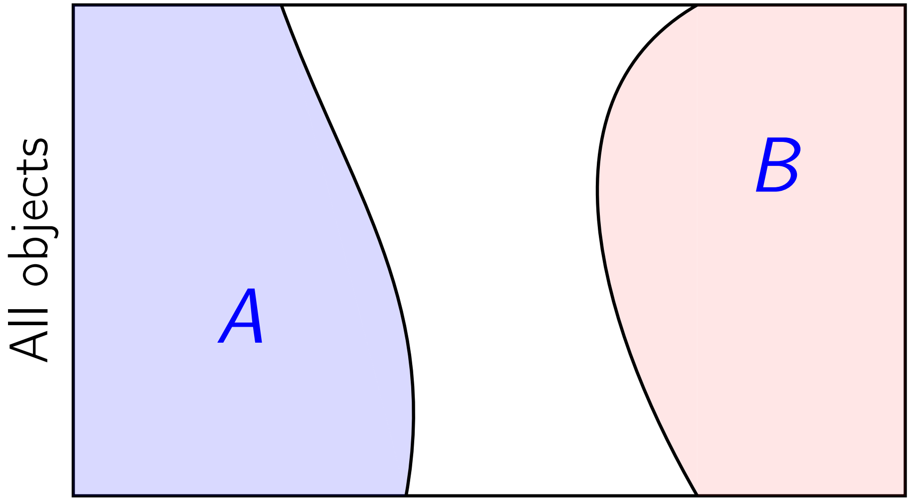
A entails **not** B

Separators

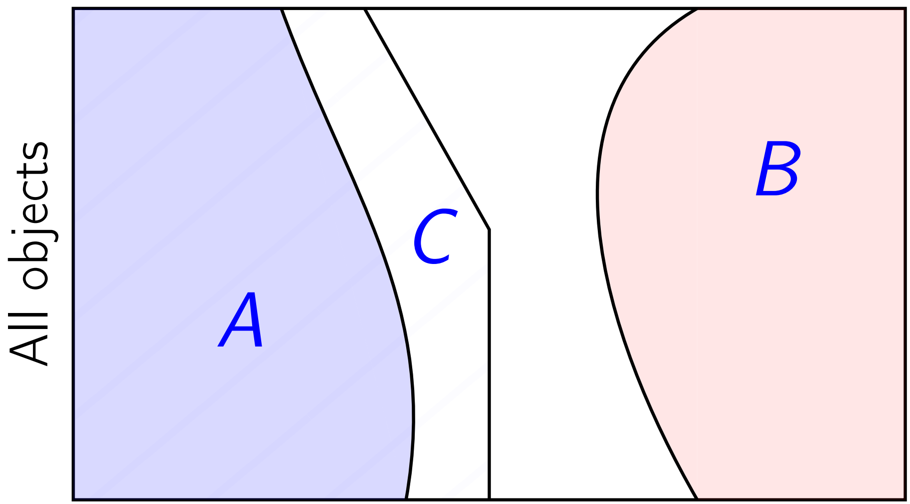


given properties A, B
which exclude each other:

A entails **not** B

a **separator** is an
intermediate property C:

Separators

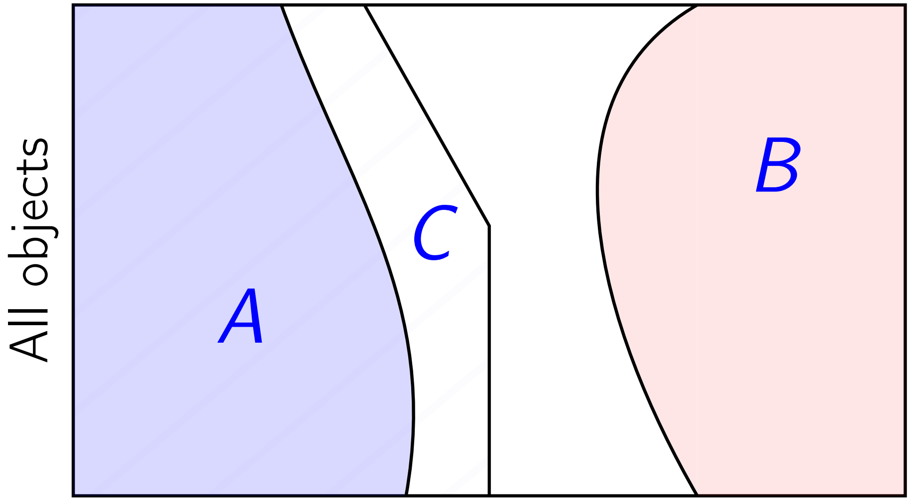


given properties *A*, *B* which exclude each other:

A entails **not** *B*

a **separator** is an intermediate property *C*:

Separators



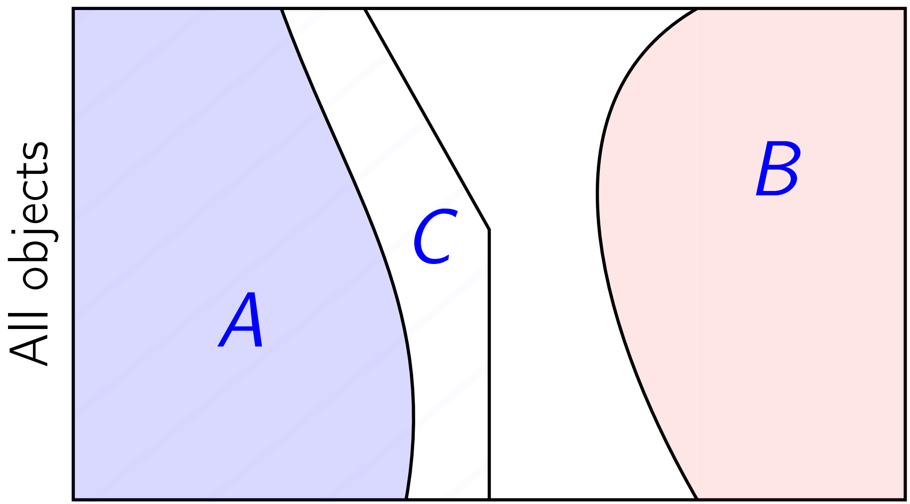
given properties A, B
which exclude each other:

A entails **not** B

a **separator** is an
intermediate property C:

A entails C and
C entails **not** B

Why (and when) is it interesting?



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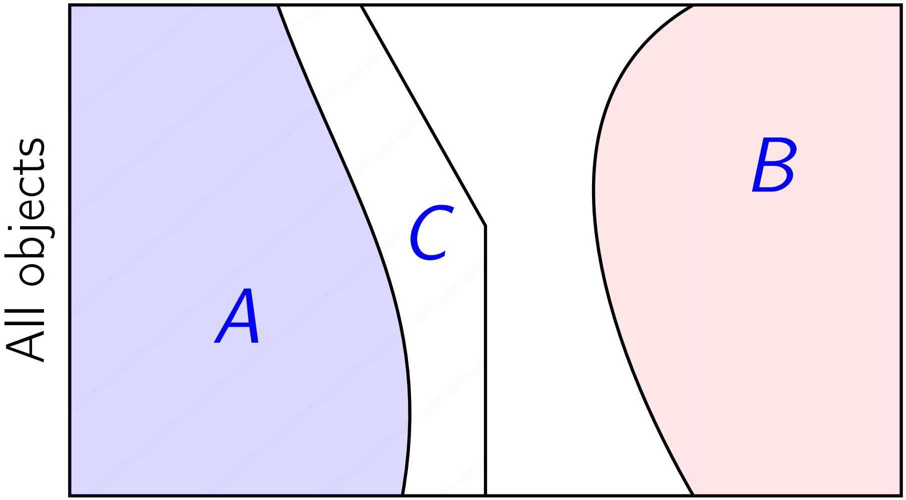

separation is interesting when C is simpler than A, B

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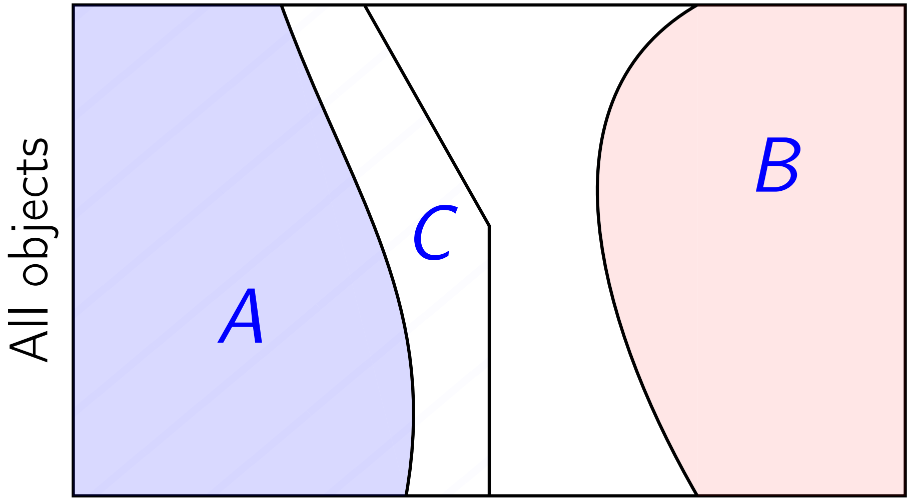

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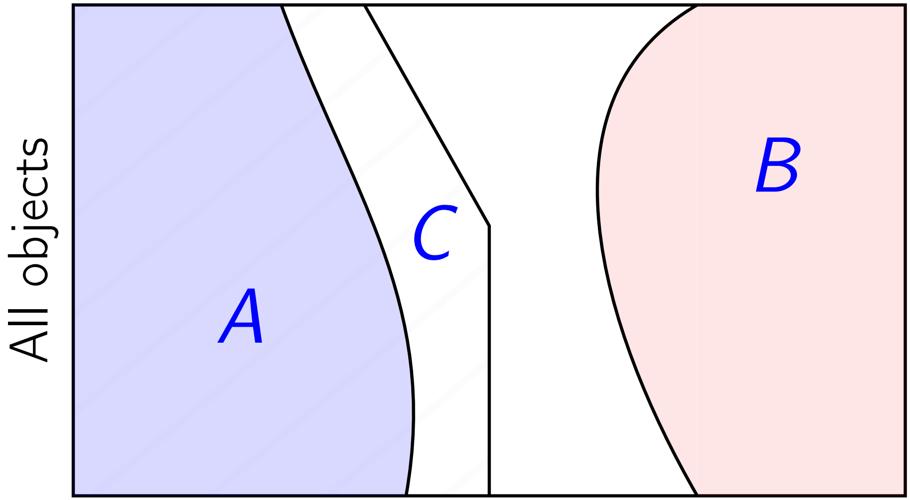

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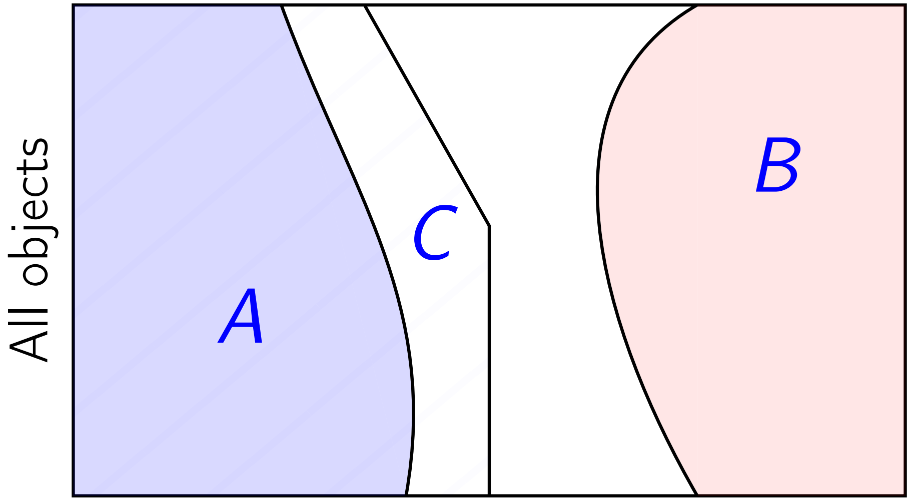

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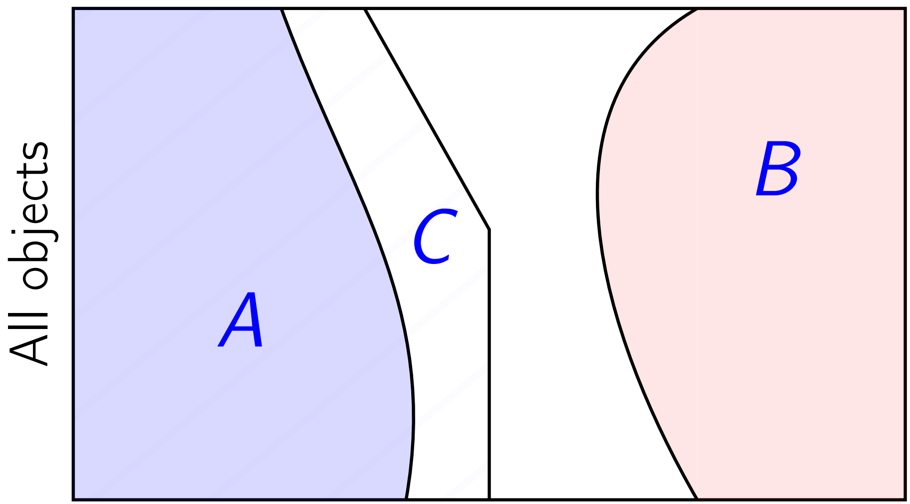

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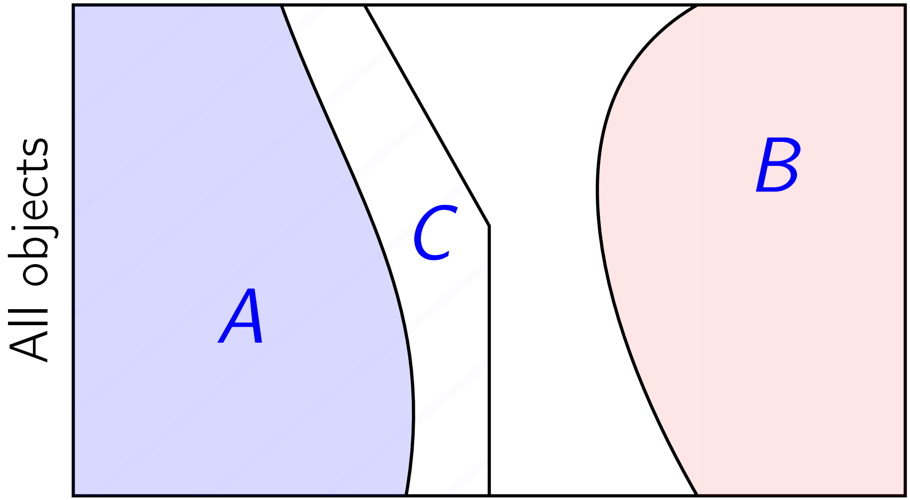

separation is interesting when C is simpler than A, B

- simple explanation of complicated contradiction
- subsumes other classical notions (definability, interpolation)
- relevant in practice e.g. for reconciling conflicting specifications

Example:

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All natural numbers

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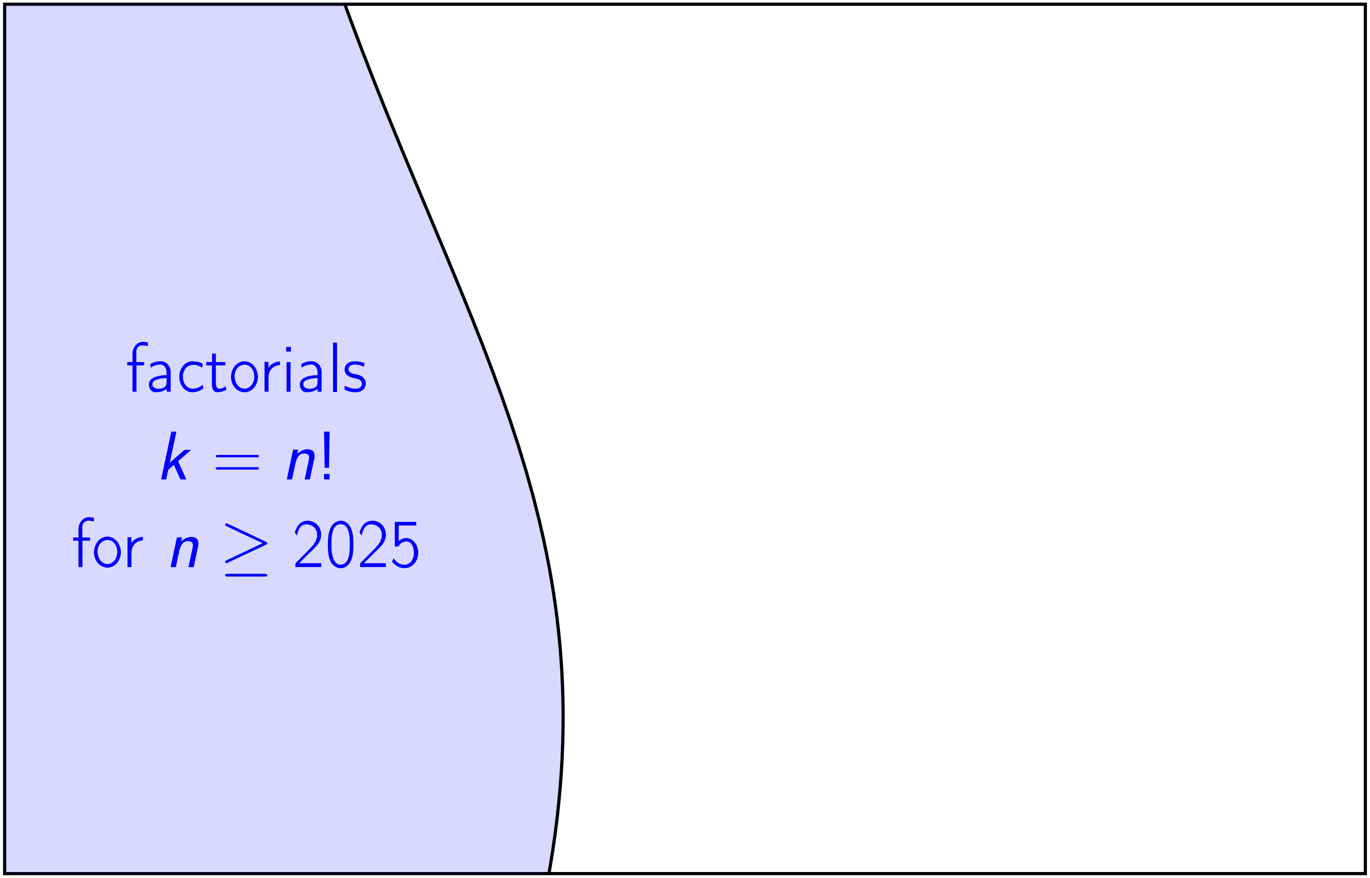

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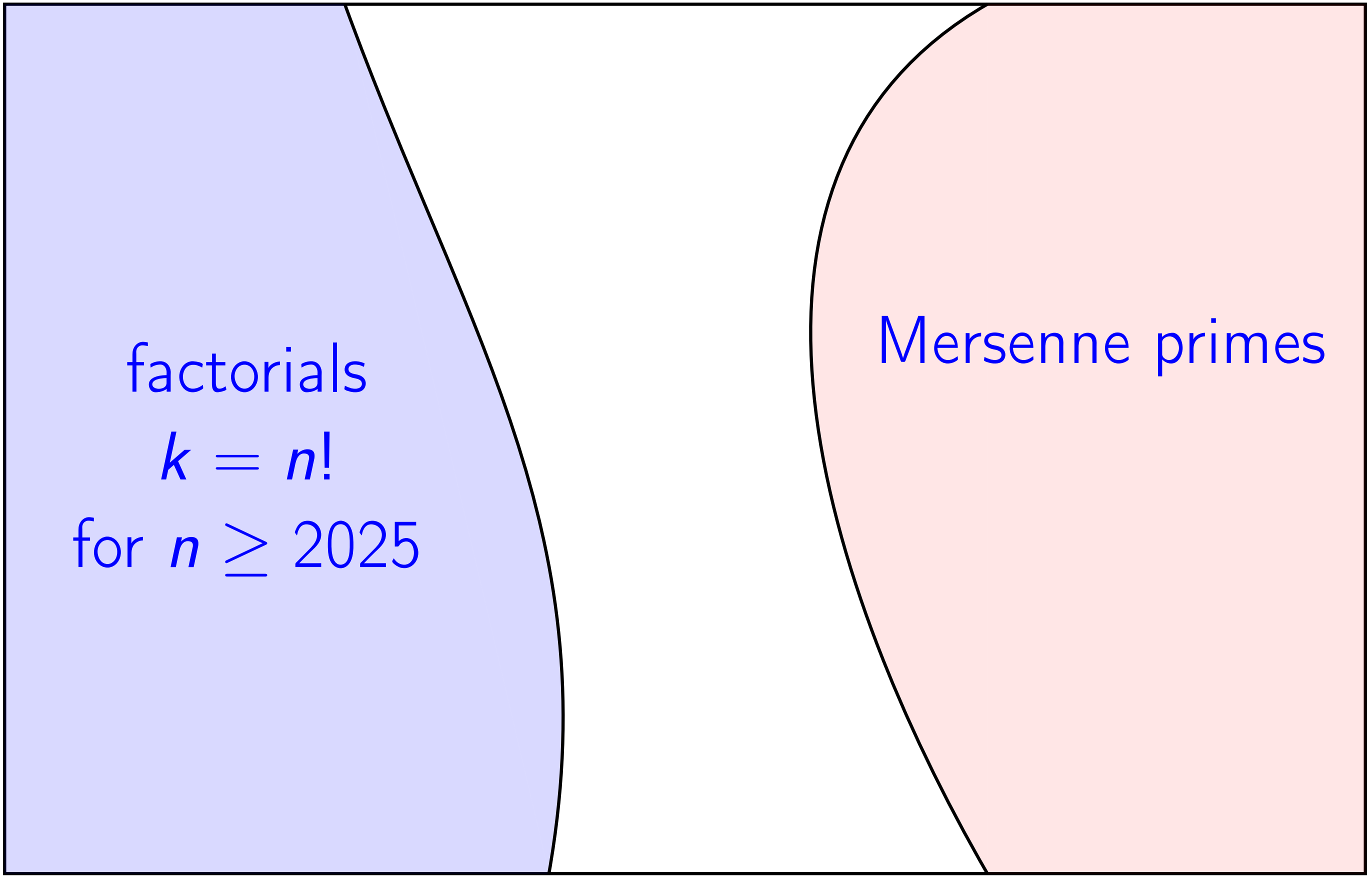

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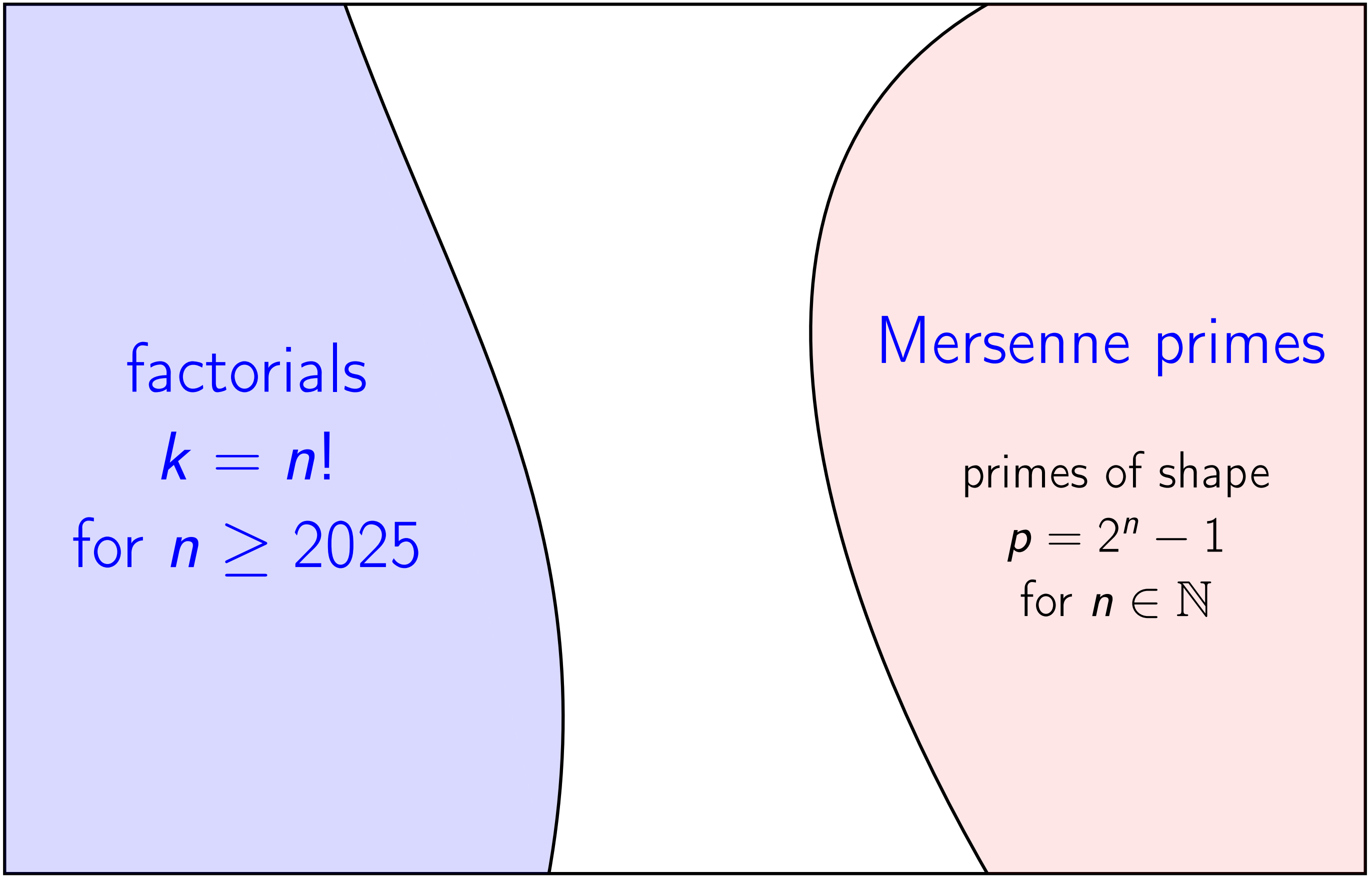
factorials
$k = n!$
for $n \geq 2025$
Mersenne primes
primes of shape
$p = 2^n - 1$
for $n \in \mathbb{N}$

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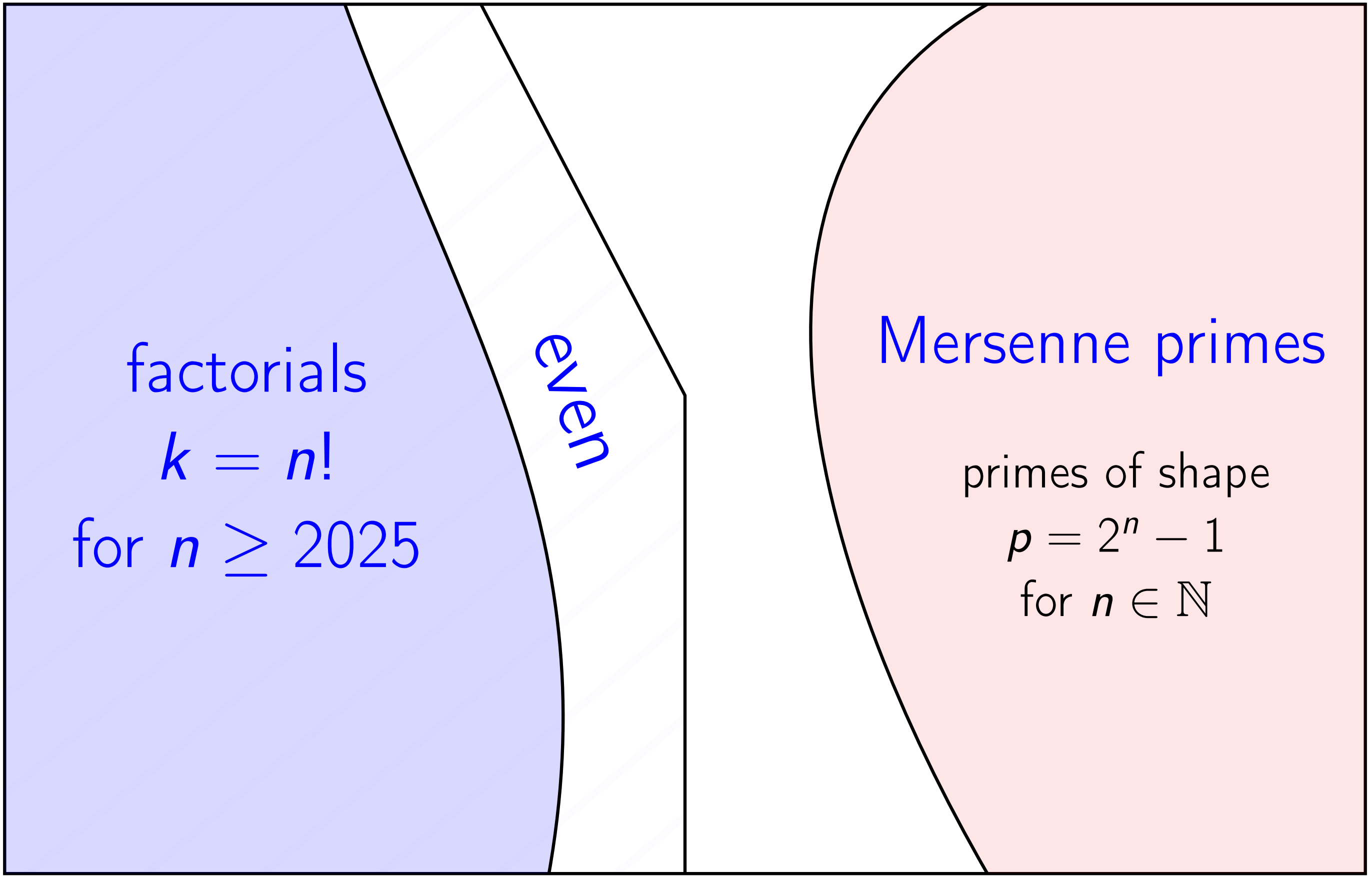
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The setting: logic

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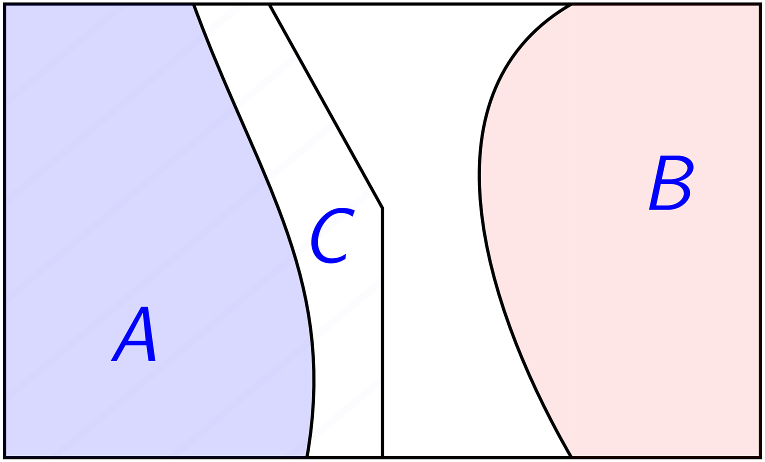

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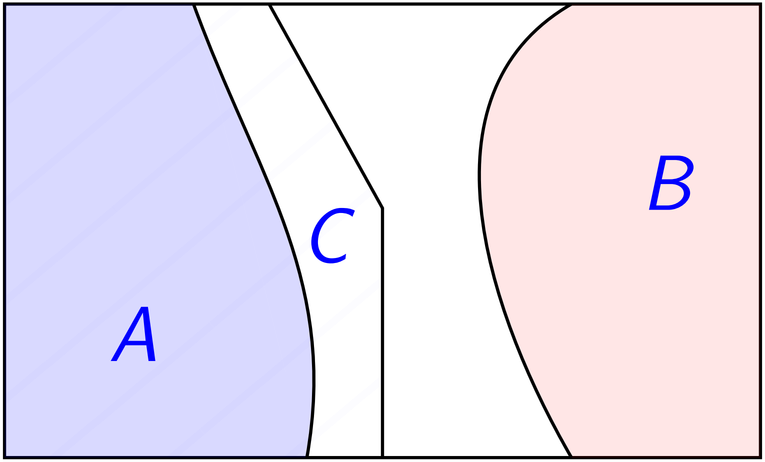

properties A,B,C specified

by **logical formulae**

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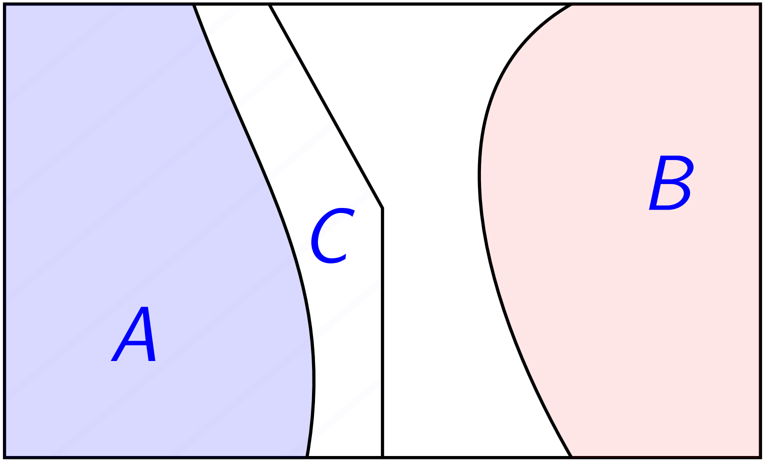

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(e.g. $\mathsf{even}(x) \iff \exists y . x = y + y$)

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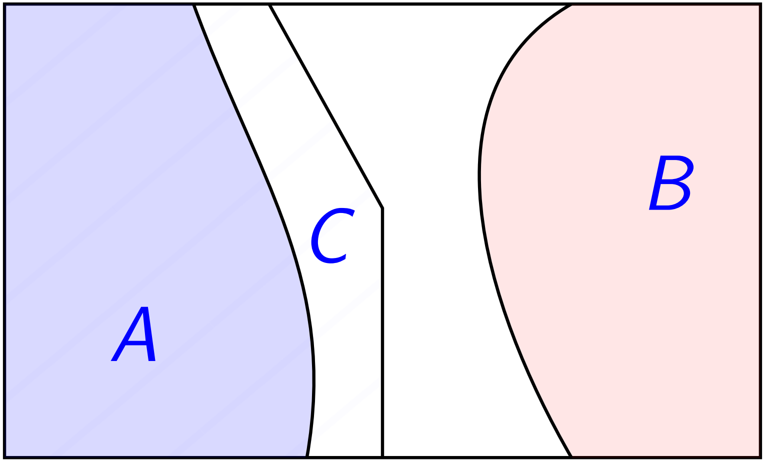

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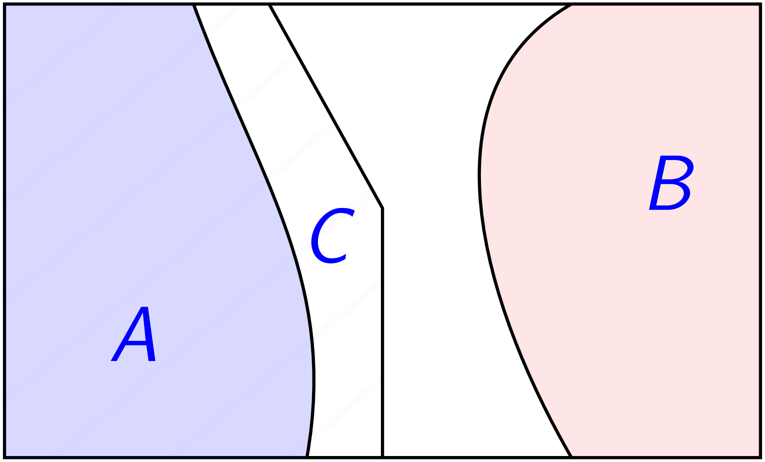

in complicated
logic $\mathcal{L}^+$

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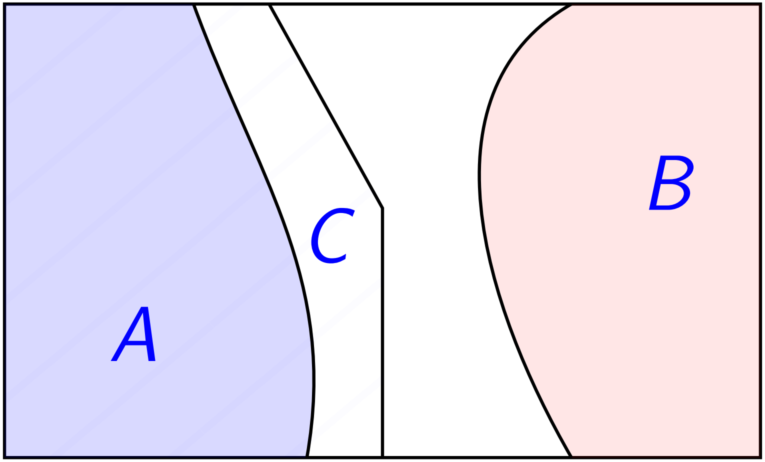

modal logic with extras, monadic second-order logic...

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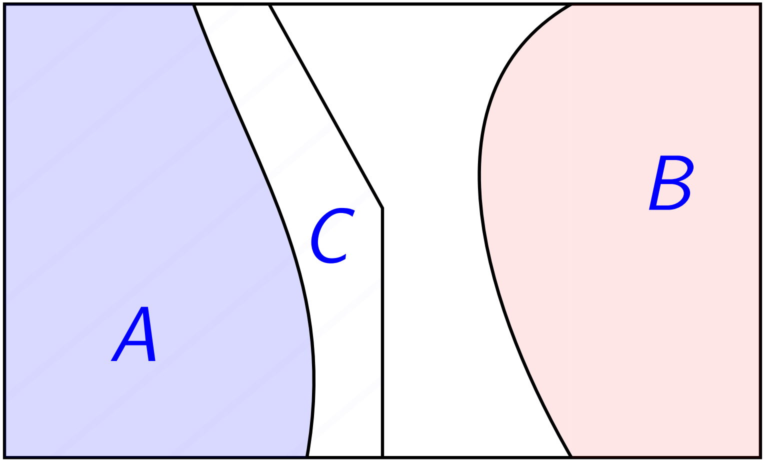

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simple C

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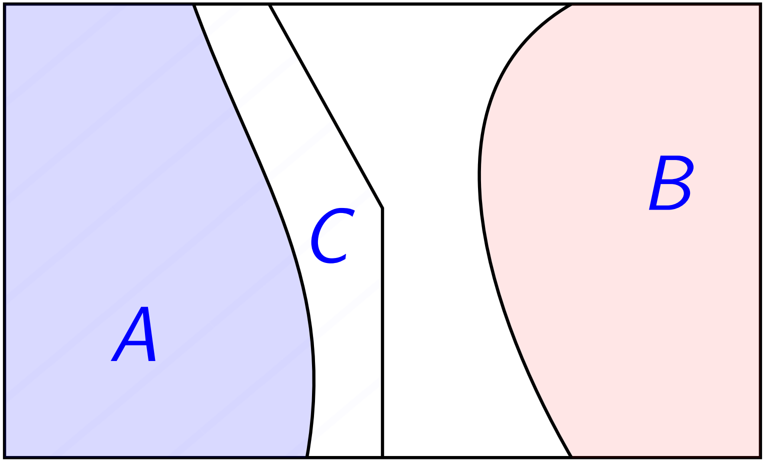

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in easy logic $\mathcal{L}$

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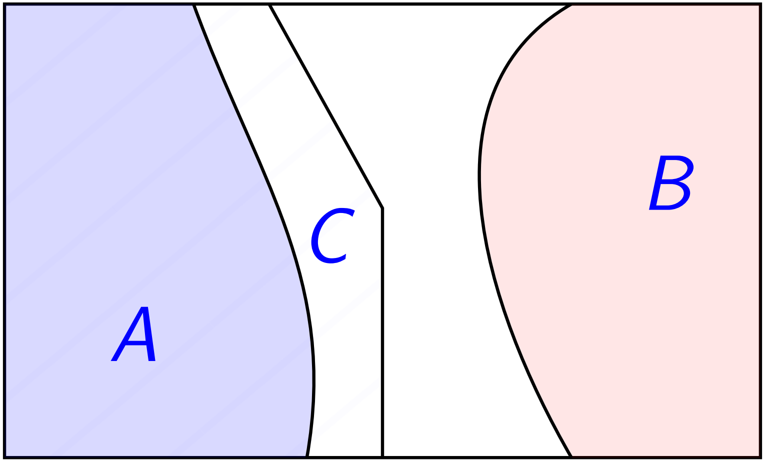

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given $A, B \in \mathcal{L}^+$

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(1) is there a separator $C \in \mathcal{L}$?

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Thank you!