



# Photonic quantum information processing

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- My short bio
- Quantum Technologies Research Group "QCAT"
- Quantum photonic platform (short overview)
- Quantum Kravchuk Transform
- Application of Quantum Kravchuk Transform to quantum simulations
- Conclusions

#### My short bio

- **2007** PhD in physics, Faculty of Physics, University of Warsaw
- 2007-2012 Max Planck Institute for the Science of Light & Erlangen-Nuremberg University
- 2012-2016 University of Gdańsk & Institute of Physics PAS FNP "Homing Plus", MSCA Career Integration Grant, NCN "Harmony", MNiSW "Iuventus Plus"
- **2015** Habilitation in Physics
- **2017** Visiting Professor at the University of Oxford
- 2017-2020 Faculty of Physics, University of Warsaw FNP "First Team"
- 2021-now Faculty of Mathematics, Informatics and Mechanics, University of Warsaw MSCA Innovative Training Network "AppQInfo" (coordinator) NCN "Sonata Bis", QuantEra "PhoMemtor"

#### Quantum Technologies Research Group "QCAT" www.stobinska-group.eu



# Short overview of the quantum photonic platform

#### **Quantum technologies**

Technology	Application
Quantum Key Distribution (QKD)	Ultimately safe generation of random cryptographic keys
Quantum Metrology	Performing enhanced-precision measurements (e.g. microscopy, detection of gravitational waves)
Quantum Computing (QC)	Decreasing the complexity of computations, performing simulations of quantum materials; machine learning
Quantum Random Number Generation (QRNG)	Generating the entropy for cryptographic purposes

Second quantum revolution: utilizing quantum technologies for practical applications. **It is happening just now!** 

#### **Quantum Integrated photonics**



- Works in room temperature
- Small form factor
- Relatively low costs
- Photons may be transmitted in fibers or in free space

#### Lasers, crystals and detectors







Coherent

Advr

Single Quantum

#### **Quantum photonic chips**



Laser micromachining



Lithography

#### **Quantum Communication (Quantum Key Distribution)**



Any "eavesdropping" of a quantum state **destroys quantum entanglement** and Alice and Bob can detect that.

For this reason, quantum key distribution is **ultimately secure**.

#### **Fiber-based Quantum Communication**



- Several companies have developed quantum key distribution systems that could enable mass manufacturing of quantum security technology
- Target audiences: governments, banking, companies
- ranges up to 800 km
- Basis of so-called "Quantum Internet"

#### **Quantum communication in space**



- In 2017 China built a satellite for longdistance quantum communication
- The distance between ground stations: between 1600 and 2400 km
- Speed: 1 photon pair every 2 seconds
- A few new EU and non-EU projects have recently emerged, and we will see more satellite-based quantum solutions

#### **Quantum metrology**



- Quantum metrology allows to bypass limits of optical resolution (Standard Quantum Limit)
- Applications: new microscopy solutions (replacing cryogenic devices in biology and medicine), enhanced precision sensing, observing gravitational waves

### **Quantum (photonic) computing**



- **Quantum computing** is nothing more than gathering statistics from the measurements of the system state. Each program must be run thousands of times.
- **Quantum processor** is controlled by a classical system. At each run it starts from a ground state and applies a series of operations (described as quantum gates) which evolve it.

#### **Photonic Quantum Machine Learning**



Xanadu's photonic processor

- Quantum Machine Learning (QML) merges classical and quantum computing techniques to achieve faster learning and lower energy consumption
- **Applications:** robotics, classification of data, computer vision

# Quantum Kravchuk Transform and its applications

#### **Publication: the Quantum Kravchuk Transform**

# Quantum interference enables constant-time quantum information processing

Science Advances 5, eaau9674 (2019)

M. Stobinska<sup>1</sup>, A. Buraczewski<sup>1</sup>, M. Moore<sup>2</sup>, W. R. Clements<sup>2</sup>, J. J. Renema<sup>3</sup>, S. W. Nam<sup>4</sup>, T. Gerrits<sup>4</sup>, A. Lita<sup>4</sup>, W. S. Kolthammer<sup>2</sup>, A. Eckstein<sup>2</sup> & I. A. Walmsley<sup>2</sup>

<sup>1</sup> University of Warsaw

<sup>2</sup> University of Oxford (now: Imperial College), UK

<sup>3</sup> University of Twente, The Netherlands

<sup>4</sup> National Institute of Standards and Technology, Boulder, Colorado, USA



#### **Publication: Quantum simulations**

#### **Quantum simulations with multiphoton Fock states**

npj Quantum Information 7, 91 (2021)

T. Sturges<sup>1,\*</sup>, T. McDermott<sup>1,\*</sup>, A. Buraczewski<sup>1</sup>, W. R. Clements<sup>2</sup>, J. J. Renema<sup>3</sup>, S. W. Nam<sup>4</sup>, T. Gerrits<sup>4</sup>, A. Lita<sup>4</sup>, W. S. Kolthammer<sup>2</sup>, A. Eckstein<sup>2</sup>, I. A. Walmsley<sup>2</sup>, M. Stobińska<sup>1</sup>

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npj quantum information

## Key motivation: help improving medical diagnostics



- Images of 512 x 512 pixels were transformed to the frequency domain
- 1% additive Gaussian noise was added
- Inverse transforms were applied
- Effective size of voxels is several millimetres!
- Neuroscience requires micrometers...

#### **General motivation**



### Motivation for performing quantum simulations

- Essential tool for studying complex phenomena: quantum topology, quantum information transfer, relativistic wave equations.
- **Primary resources used so far:** collections of qubits, coherent states, multiple single-particle Fock states.
- Quantum simulations have never seriously profited from interference of multiparticle Fock states. [F. Flamini et al. Rep. Prog. Phys. 82, 016001 (2019).]



#### **Our achievements (theory + experiment)**

#### Multiphoton Fock state interference

- computes the fractional Kravchuk transform in a single step
- has provided a resource-efficient basic demonstration that has revealed a topological matter not known before,
- can simulate non-linear systems,
- has elucidated the perfect quantum wave packet transfer mechanism and transport of Majorana fermions

We work within the quantum photonics framework, but we **generalize our results** beyond it.

#### Fourier Transform (FT)



### **Discrete Fourier Transform (DFT)**

A discrete approximation of the FT (discretization):

$$X_k = \frac{1}{\sqrt{S+1}} \sum_{l=0}^{S} e^{-2\pi i \cdot \frac{kl}{S+1}} \cdot x_l$$



 $(x_0, x_1, \dots, x_S) \to (X_0, X_1, \dots, X_S)$ 

- DFT  $\longrightarrow$  FT for large S
- Number of operations:  $O(n^2)$ , n = S + 1
- Fractional DFT do not reproduce fractional FT
- FFT computes DFT in  $O(n \log n)$  for periodic data with period  $n = 2^m$

#### **Fast Fourier Transform (FFT)**



- Computes the DFT with a "divide & conquer" method
- Operates on sequences of  $S = 2^n$  length (zero padding introduces errors)
- Lowers the number of operations from  $O(2^{2n})$  to  $O(n2^n)$

[J. W. Cooley et al., IEEE Trans. on Audio and Electroacoustics 15, 76 (1967)]

#### **Fractional FT**

a-power of FT,  $\mathcal{F}^{\alpha}{f(t)}$ , corresponds to a rotation by  $\frac{\pi\alpha}{2}$  in the phase space  $\mathcal{F}^{0}{f(t)} = f(t)$   $\mathcal{F}^{1}{f(t)} = F(\omega)$  $\mathcal{F}^{2}{f(t)} = f(-t)$   $\mathcal{F}^{3} = \mathcal{F}^{-1}$ 

**Tomography of quantum states:** fractional FT is the Radon transform of the Wigner distribution

$$\mathcal{R}_{\alpha\pi/2}\{W\} = |\mathcal{F}^{\alpha}\{\psi(t)\}|^2$$

[P.-Y. Lin, *The Fractional Fourier Transform and Its Applications*, National Taiwan University, Taipei, Taiwan (1999)]



#### **Fractional FT – the main applications**

- Processing of noisy data
- Exploring a specific region of interest
- Optical signal processing

- Phase retrieval
- Tomography
- Data compression

[A. Camara et al., J. Opt. Soc. Am. A 26, 1301 (2009)]



#### **Quantum Fourier Transform (QFT)**



Performs the DFT on quantum amplitudes with quantum gates

$$|X_k\rangle = \frac{1}{\sqrt{S+1}} \sum_{l=0}^{S} e^{2\pi i \cdot \frac{kl}{S+1}} |x_l\rangle$$

• Lowers the number of operations from  $O(n2^n)$  to  $O(n \log n)$ 

#### **Kravchuk Transform (KT)**

Alternative discrete approximation of the FT

$$X_k = \sum_{l=0}^{S} e^{i\vartheta_{kl}} \cdot \phi_k^{(p)}(l - Sp, S) \cdot x_l$$

$$p = \sin^2 \frac{\pi \alpha}{4}$$
$$\vartheta_{kl} = -\frac{\pi \alpha}{2} \frac{S}{2} + \frac{\pi}{2}(l-k)$$

- Works for non-periodic data of arbitrary length
- Utilizes Kravchuk functions  $\phi_k^{(p)}(l', S)$  instead of plane waves
- DFT  $\longrightarrow$  FT for large *S* & computes  $\alpha$ -power of the FT
- Number of operations::  $O(n^2)$ , n = S + 1
- Can be viewed as an overlap of two spin S/2 states

$$e^{i\frac{\pi}{2}(l-k)}\phi_k^{(p)}(l-Sp,S) = \left\langle \frac{S}{2}, \frac{S}{2} - k \right| \exp\left(i\frac{\pi\alpha}{2}S_x\right) \left|\frac{S}{2}, \frac{S}{2} - l\right\rangle$$

# Kravchuk functions $\phi_n^{(p)}(n', N)$

• **Physically meaningful:** they describe eigenstates of a finite harmonic oscillator



- For  $N \to \infty$  they tend to Hermite-Gauss polynomials (quantum harmonic oscillator)
- They are **discrete orthogonal polynomials** associated with the binomial distribution [N. M. Atakishiev, K. B. Wolf, J. Opt. Soc. Am. A **7**, 1467 (1997)]

## Kravchuk functions $\phi_n^{(p)}(n', N)$ (cont'd)

Kravchuk functions obey the following difference equation

$$\left(\frac{1}{2}N - n\right)\phi_n^{(1/2)}(n', N) = \frac{1}{2} \left[\alpha(n')\phi_n^{(1/2)}(n' - 1, N) + \alpha(n' + 1)\phi_n^{(1/2)}(n' + 1, N)\right]$$
$$\alpha(n') = \left[\left(\frac{1}{2}N + n'\right)\left(\frac{1}{2}N - n' + 1\right)\right]^{\frac{1}{2}},$$

which can be written in a form of an **eigenvalue equation**  $\left\{-\frac{1}{2}[\alpha(n')\exp(-\partial_{n'}) + \alpha(n'+1)\exp(\partial_{n'})] + \frac{1}{2}(N+1)\right\}\phi_n^{(1/2)}(n',N) = \left(n + \frac{1}{2}\right)\phi_n^{(1/2)}(n',N)$   $\exp(x\partial_{n'})f(n') = f(n'+x) - \text{a finite-shift operator}$ 

## Kravchuk functions $\phi_n^{(p)}(n', N)$ (cont'd)

The eigenequation possesses a real, equally-spaced spectrum characteristics of the **harmonic oscillator** 

$$\boldsymbol{H}^{(N)}(s) \,\phi_n^{(1/2)}(s,N) = \left(n + \frac{1}{2}\right) \phi_n^{(1/2)}(s,N), \qquad n = 0, 1, \dots, N$$

where

$$H^{(N)}(s) = -\frac{1}{2} [\alpha(s) \exp(-\partial_s) + \alpha(-s) \exp(\partial_s)] + \frac{1}{2} (N+1).$$

One may identify  $H^{(N)}(s)$  as the Hamiltonian of a finite quantum harmonic oscillator.

This Hamiltonian is related to the Lie algebra of the rotation group SO(3).

#### **Fractional Fourier-Kravchuk Transformations**

We define the  $\alpha$ th power of the one-dimensional finite Fourier–Kravchuk transform  $\hat{F}$  by a linear operator that maps the Kravchuk basis functions onto themselves:

$$\widehat{F}^{\alpha} \phi_n(s, N) = \exp\left(\frac{i\alpha\pi}{4}\right) \exp\left[-\frac{i\pi}{2}\alpha H^{(N)}(s)\right] \phi_n(s, N)$$
$$= \exp\left(-\frac{i\alpha\pi n}{2}\right) \phi_n(s, N)$$

Properties:

- $F^0 = 1$ ,
- $F^4 = 1$ ,
- its square is the inversion matrix:  $F^2 = I$ ,
- $F^{\alpha}$  are unitary:  $(F^{\alpha})^{\dagger} = F^{-\alpha}$ .

#### **The Fourier-Kravchuk Rotations**

$$[J_1, J_2] = iJ_3, \qquad [J_2, J_3] = iJ_1, \qquad [J_3, J_1] = iJ_2$$

#### where

$$J_{1} = s \cdot,$$
  

$$J_{2} = -P_{s}^{(N)}, \qquad [s, \mathbf{H}^{(N)}(s)] = \frac{1}{2} [\alpha(-s) \exp(\partial_{s}) - \alpha(s) \exp(-\partial_{s})] = iP_{s}^{(N)}$$
  

$$J_{3} = \mathbf{H}^{(N)}(s) - \frac{1}{2} (N+1).$$

#### Finite signal processing – DFT vs. KT



## **KT** – exemplary applications

 Image analysis with the Kravchuk moments – transform coefficients are used as data vectors for shape recognition

[P. T. Yap et al., IEEE Transactions on image processing 12, 1367 (2003)]

- Reconstruction of medical images (MRI, ultrasound), especially for high resolution data
- Automatic identification of tumors in computer tomography scans
- Character recognition (e.g. Chinese)
- Error-correcting codes
- Digital watermarking, anti-fraud techniques (fractional KT)

KT features a prohibitive runtime

#### Photonic waveguide realization of the QKT



- Optical chip with parallel planar waveguides
- The fractionality is set by the chip length
- The input and output sequence length is limited by the number of waveguides
- Requires S photon sources and detectors

[N. M. Atakishiev, K. B. Wolf, J. Opt. Soc. Am. A 7, 1467 (1997),
 S. Weimann et al., Nature Commun. 7, 11027 (2016),
 A. Crespi et al., Nature Commun. 7, 10469 (2016)]

#### **Our result: QKT in a single step**

Multiphoton Hong-Ou-Mandel interference computes the QKT in O(1)



#### **Two-photon HOM effect**



Changing the BS reflectivity alters the output distribution



#### **Multiphoton generalized HOM effect**



#### **Practical steps 1**

Let's send a superposition to a BS

$$|\psi\rangle = \sum_{l=0}^{S} x_{l} |l, S - l\rangle$$

BS interaction is photon number conserving

$$U_{BS}(\theta, \varphi) = \exp\left\{\frac{\theta}{2}\left(a^{\dagger}be^{-i\varphi} - ab^{\dagger}e^{i\varphi}\right)\right\}$$
$$r = \sin^{2}\frac{\theta}{2} \text{ is the BS reflectivity (we take } \varphi = \frac{\pi}{2}).$$
Let's use photon-number-resolved measurements (TESs) – projective measurement

$$|k\rangle\langle k| \& |S-k\rangle\langle S-k|$$

#### **Practical steps 2**

Probability of measuring k and S - k at outputs is equal to

$$\left|\langle k, S-k|U_{BS}\left(\theta, \frac{\pi}{2}\right)|\psi\rangle\right|^{2} = \left|\sum_{l=0}^{S} x_{l} \langle k, S-k|U_{BS}\left(\theta, \frac{\pi}{2}\right)|l, S-l\rangle\right|^{2}$$

gives the absolute square of the QKT of input probability amplitudes

$$|X_k|^2 = \left|\sum_{l=0}^{S} \mathcal{A}_S^{(r)}(k,l) x_l\right|$$

2

QKT fractionality is set by the reflectivity  $\alpha = \frac{2\theta}{\pi} = \frac{4}{\pi} \arcsin \sqrt{r}$ .

#### Generalization to qudits

#### Multiphoton HOM interference realizes a single-qudit rotation



- Dual Fock states with S photons (blue) map onto spin-S/2 Dicke states (black)
   – qudit encoding
- BS interaction models exchange interaction
- This coincides with a qudit rotation

$$R_{\theta,\varphi=\pi/2} = \exp\{-i\theta S_x\} = U_{BS}\left(\theta,\varphi=\frac{\pi}{2}\right)$$

in the Dicke state basis

#### **QKT** as a single-qudit rotation



Bloch sphere:

the QKT transfers the input – a position eigenstate – into the same state but in  $S_y$  basis – a momentum eigenstate

### **Experimental setup (University of Oxford)**



- SPDC spontaneous parametric down-conversion waveguides in PP-KTP crystal.
- **WP** Wollaston prism.
- VC variable-ratio fiber coupler.
- TES transition-edge sensors with efficiency exceeding 90% → photon-number resolved measurement.
- **DAQ** data acquisition unit.

#### **Experimental setup (University of Oxford)**



$$|0, S\rangle$$
 interference –  $(x_0 = 1, x_2 = 0, ..., x_S = 0)$ 

![](_page_46_Figure_1.jpeg)

$$l, S - l$$
 interference –  $(x_0 = 0, ..., x_l = 1, ..., x_S = 0)$ 

![](_page_47_Figure_1.jpeg)

#### Conclusions

- Realization of the fractional QKT with qudit systems shows that transformation of large data sequences in O(1) is possible
- Since a BS sees orthogonal spectral or polarization modes independently, one can extend the transform to higher dimensions
- The photonic proof of concept is currently limited by the range of input states that can be prepared
- New applications: studying non-crystalline topological materials, beyond the recently challenged bulk-edge correspondence theorem. [C. Downing et al., Phys. Rev. Lett. 123, 217401 (2019)].
- Qudit-based algorithms exhibit significantly lower complexity than qubit-based ones

#### Patent

#### A Method of Performing Quantum Fourier-Kravchuk Transform (QKT) and a Device Configured to Implement Said Method

![](_page_49_Picture_2.jpeg)

![](_page_49_Picture_3.jpeg)

- Patent Office of the Republic of Poland (2018) PL426228
- World Intellectual Property Organization (WIPO) (2019) WO/2020/008409
- US Patent Office (2021) US20210271731
- China Patent Office (2021) CN113692593

#### Lecture of Prof. Philip Walther

- All interested in the topic are kindly invited to a seminar, which will be given by Prof. Philip Walther

   a renowned quantum physicist from the University of Vienna.
- Title: Quantum Photonics from quantum computing to quantum foundations exploring the quantum-gravity interface
- Place: Room 2180, MIM UW building (Banacha 2)
- Date/time: Thursday, 3 November 2022, 14:30 hours

![](_page_50_Picture_5.jpeg)

#### Thank you!

![](_page_51_Picture_1.jpeg)

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www.stobinska-group.eu