OPERATOR SEMIGROUPS IN THE CALKIN ALGEBRA (PART 2)

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In the second part of the talk, continuing our discussion concerning continuity of semigroups in the Calkin algebra $\mathcal{Q}(\mathcal{H})$, we shall present a sketch of proof of the characterization of SOT-continuity of a dyadic semigroup $(q(t))_{t\in\mathbb{D}}\subset\mathcal{Q}(\mathcal{H})$ induced by the zero element of the extension group $\mathrm{Ext}(X)$, where X is an admissible compact metric space. Namely, if $X=\varprojlim X_n$, where $X_n=\overline{\exp(2^{-n}Z)}$, $Z\subset\mathbb{C}$ is a closed set lying in some left half-plane, and $\pi_n\colon X\to X_n$ stands for the n^{th} projection, for each $n\in\mathbb{N}$, then for the semigroup $(q(t))_{t\geq 0}$ induced by $\Theta\in\mathrm{Ext}(X)$ the following conditions are equivalent:

(i) $(q(t))_{t\in\mathbb{D}}$ is strongly continuous with respect to a fixed Calkin's representation $\gamma\colon\mathcal{Q}(\mathcal{H})\to\mathcal{B}(\mathbb{H})$;
(ii) $(q(t))_{t\in\mathbb{D}}$ is strongly continuous with respect to all Calkin's representations;
(iii) $\lim_{n\to\infty}\pi_n(\xi)=1$ for every $\xi\in X$.

Next, we shall discuss the lifting problem for C_0-semigroups $(q(t))_{t\geq 0}$ in $\mathcal{Q}(\mathcal{H})$, i.e. the question whether one can find a (strongly continuous) operator semigroup $(Q(t))_{t\geq 0}$ in $\mathcal{B}(\mathcal{H})$ such that $\pi Q(t)=q(t)$ for $t\geq 0$. Let A be the generator of $(q(t))_{t\geq 0}$. By using Milnor's exact sequence, we show that if each $q(t)$ has a normal lift, then the question whether the extension Γ induced by $(q(t))_{t\geq 0}$ is trivial reduces to the question whether the corresponding first derived functor $\varprojlim{}^{(1)}\,\mathrm{Ext}_2(X_n)$ for the suspensions of $X_n=\overline{\exp(2^{-n}\sigma(A))}$ vanishes. With the aid of the CRISP property and Kasparov's Technical Theorem, we propose two results which provide some geometric conditions on $\sigma(A)$ guaranteeing splitting of Γ.

The talk will be mostly based on the preprint: *Compact perturbations of operator semigroups*, `arXiv:2203.05635v2`

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