KKM Theorem: From L*-operators to a Renting Problem.

An $L^{*}-$*operator* on a topological space X is any function $\Lambda : [X]^{<\omega} \to \exp(X)$ satisfying the following condition: $(*)$ If $A \in [X]^{<\omega}$ and $\{U_x : x \in A\}$ is an open cover of $X,$ then there exists a $B \subseteq A$ such that $\Lambda(B) \cap \bigcap \{U_x : x \in B\} \neq \emptyset$. The convex hull operator is an $L^{*}-$*operator* (it is a disguised version of the celebrated KKM theorem), which is $n-$ continuous for each $n = 1, 2, \ldots$. Any metrizable continuum that admits a $2-$continuous $L^{*}-$*operator* must be locally connected and unicoherent. Compact spaces that admit continuous $L^{*}-$*operator* have the fixed point property.

A statement of the colorful version of the KKM theorem has been modelled on the celebrated Barany's colorful Caratheodory's theorem. It holds true only for some special KKM families. Within the scope of its applications, we show how to get a fair allocation in a situation involving a finite number of individuals, the same number of indivisible goods, and one divisible good.

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Institute of Mathematics, Card. S. Wyszyński University, Warsaw a.szymanski@uksw.edu.pl,