Johnson-Schechtman disjointification inequalities for U-statistics with application to interpolation theory and biparameter martingale inequalities

A classical inequality of Rosenthal allows to express, up to a constant dependent only on p, the p-th moment ($p \geq 1$) of a sum of independent nonnegative random variables in terms of moments of their disjoint sum. There is a counterpart to this ineqaulity for $0 < p < 1$ due to Johnson and Schechtman. We present an extension of the latter to nonnegative generalized U-statistics. This turns out to translate directly into results about interpolation of spaces spanned by canonical generalized U-statistics in $L^1(\ell^p)$. We also use it to show that the L^1 norm of a square function of a double-indexed martingale dominates the L^1 norm of its maximal function.