Differential equations with coefficients in Sobolev spaces.

I would like to propose a model of ordinary and delay differential equations with coefficients in Sobolev spaces. These are equations of the form

$$y'(t) = f(t, y(t)) \quad , \quad y'(t) = f(t, y(t), y(t-1))$$

where f is a discontinuous, measurable and locally integrable function.

An example of a problem with unbounded discontinuous right-hand side is the n-body problem

$$c_i''(t) = \gamma \cdot \sum_{1 \leq j \leq n, i \neq j} m_j \cdot \frac{c_j(t) - c_i(t)}{\|c_j(t) - c_i(t)\|^3}$$

where: the mass of the i-th body is m_i and $c_j(t)$ is the position vector in R^3 of the j-th body. Moreover, γ is the gravitiational constant; $c_i''(t)$ is the acceleration of i-th body. If these bodies collide, then the right-hand side becomes singular.

The Caratheodory conditions for $f(t, x)$ guarantee the existence of solutions to problem $y'(t) = f(t, y(t))$. These conditions are that f is measurable with respect to t and continuous (or Lipschitz continuous) with respect to x.

When we study differential equations with measurable coefficients, then there are some obstacles. Briefly speaking, if function $f(t, x)$ is at most measurable, then the superposition $f(t, y(t))$ does not have to be measurable and we lose the meaning of the integral $\int_0^t f(s, y(s))ds$.

I would like to propose a generalized meaning of the above integral: this might be the concept of the trace operator of function $f(t, x)$ on the graph of curve $y(t)$.