## Computational Social Choice and Fair Participatory Budgeting

Piotr Skowron
University of Warsaw


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Studying situations where a group must make a decision, yet the members of the group have contradictory preferences regarding the outcome.

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## VOTE



## Preferences

decision
Outcome

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Preferences
decision
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## The model for participatory budgeting

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1. A set of candidates or projects $C=\left\{c_{1}, c_{2}, \ldots, c_{m}\right\}$.

Each candidate $c$ comes with a cost, $\operatorname{cost}(c)$.
2. There is a budget constraint $b$ : We have to select a subset of projects $W$ s.t. $\sum_{c \in W} \operatorname{cost}(c) \leq b$.
3. A set of voters $N=\{1,2, \ldots, n\}$.

Each voter has preferences over the projects.

## How this is currently done



## How this is currently done



## How this is currently done



## How this is currently done

Solution: Divide the budget upfront between the districts!


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But this causes other problems!


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parents who want a playground

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parents who want a playground

voters close to the border

## How this is currently done

Solution: Divide the budget upfront between the districts!

But this causes other problems!

parents who want a playground

voters close to the border

cyclists who want a bike trail

## How this is currently done



Districts are not the only division of voters

## How this is currently done

せ. \begin{tabular}{c}
$30 \%$ voters <br>
(green areas)

 

$30 \%$ voters <br>
(playgrounds)

 

$40 \%$ voters <br>
(bike infrastructure)
\end{tabular}

## How this is currently done




Choosing by the number of votes

## How this is currently done

$30 \%$ voters

(green areas) $\mathrm{l}^{30 \%}$ voters (playgrounds) | $40 \%$ voters |
| :---: |
| (bike infrastructure) |



The rule should be fair to all groups of voters

## Criterion of fairness．

| voter |  |
| :---: | :---: |
| 入ob | $170 €$ |
| $\square{ }^{\circ}$ | $25 €$ |
| X 見 | $124 €$ |
| $\square$－－ | $93 €$ |
| $\square$ | $74 €$ |
| 年 | $155 €$ |
| $x$ | $130 €$ |

$A(i)$ ：a subset of projects that voter $i$ approves．

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## Extended justified representation (EJR):

We say that a group of voters $S$ is $T$-cohesive for $T \subseteq C$ if

$$
\frac{\operatorname{cost}(T)}{|S|} \leq \frac{b}{n} \text { and } T \subseteq \bigcap_{i \in S} A(i)
$$

A rule $\mathscr{R}$ satisfies extended justified representation if for each election instance $E$ and each $T$-cohesive group $S$ of voters there exists a voter $i \in S$ such that

$$
|A(i) \cap \mathscr{R}(E)| \geq|T|
$$

$A(i)$ : a subset of projects that voter $i$ approves.

## Criterion of fairness.

| voter |  |
| :---: | :---: |
| X | $170 €$ |
| $\square \Psi^{\square}$ | $25 €$ |
| X 目 | $124 €$ |
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$$

10 voters: 思 $b=500$

10 voters:


10 voters: N-1
10 voters:
10 voters:


## Criterion of fairness.

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| :---: | :---: |
| X | $170 €$ |
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| X 6 | $170 €$ |
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Exterided' jusified represemtation ([1]in): Core:
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## Criterion of fairness.

1. K. Munagala, Y. Shen, K. Wang, Z. Wang. Approximate Core for Committee Selection via Multilinear Extension and Market Clearing. SODA-2022.
2. Z. Jiang, K. Munagala, and K. Wang. Approximately stable committee selection. STOC-2020.
3. D. Peters and P. Skowron. Proportionality and the limits of welfarism. ACM-EC-2020.
4. Y. Cheng, Z. Jiang, K. Munagala, and K. Wang. Group fairness in committee selection. ACM-EC-2019.
5. M. Brill, P. Golz, D. Peters, U. Schmidt-Kraepelin, and K. Wilker. Approval-based apportionment. AAAI-2020.
6. G. Pierczyński, P. Skowron, and D. Peters. Proportional participatory budgeting with additive utilities. NeurIPS-2021.


## Method of Equal Shares: Idea



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## Method of Equal Shares for Approvals

1. The budget is evenly divided among the voters.
2. If a candidate $c \in C$ is selected its cost is divided among the voters who voted for $c$.
3. The rule selects the projects which can be paid this way, starting with those that minimise the voters' marginal costs per utility.
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| $b=\$ 1000$ | (20 votes) | (26 votes) | (11 votes) | (9 votes) | (20 votes) | 14 votes) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A (costs \$200) | $A \checkmark$ | $A \checkmark$ |  |  |  | $\mathrm{A} \square$ |
| B (costs \$200) | В $\checkmark$ | B $\checkmark$ |  |  |  |  |
| C (costs \$200) | C $\downarrow$ | C $\checkmark$ |  | C $\checkmark$ |  | C |
| D (costs \$200) | D $\checkmark$ | D $\downarrow$ | $\mathrm{D} \checkmark$ | $\mathrm{D} \checkmark$ |  | D |
| E (costs \$200) | $\mathrm{E} \quad \checkmark$ | E $\sqrt{ }$ | E |  | E | E |
| F (costs \$200) | $\mathrm{F} \checkmark$ |  |  | $F \square$ |  | F $\checkmark$ |
| G (costs \$200) |  | G | $\mathrm{G} \square$ | $\mathrm{G} \quad \checkmark$ |  | G $\sqrt{ }$ |
| H (costs \$200) |  |  |  |  | $\mathrm{H} \square$ |  |

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A (costs \$ 200 ) | $A \checkmark$ | $A \checkmark$ |  |  | A | $A \square$ |
| B (costs \$200) | В $\checkmark$ | В $\downarrow$ |  |  |  |  |
| C (costs \$200) | C $\downarrow$ | C $\checkmark$ |  | $\mathrm{C} \square$ |  |  |
| D (costs \$200) | D $\downarrow$ | D $\downarrow$ | $\mathrm{D} \square$ | $\mathrm{D} \square$ | D | D |
| E (costs \$200) | $\mathrm{E} \quad \checkmark$ | $\mathrm{E} \quad \checkmark$ | E |  | E |  |
| F (costs \$200) | $\mathrm{F} \square$ |  |  | $F \square$ |  | $F \longrightarrow$ |
| G (costs \$200) |  | G | $\checkmark$ | $\mathrm{G} \quad \checkmark$ |  | $\mathrm{G} \quad \checkmark$ |
| H (costs \$200) |  |  |  |  | H |  |
|  | 20 voters: | 26 voters: | 11 voters: | 9 voters: | 20 voters: | 14 voters: |
| \$10 | 3.33 | 3.33 |  |  |  |  |
| $\downarrow$ | 3.03 | 3.03 | 3.03 | 3.03 |  | 3.33 |

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A (costs \$200) | $A$ | $A$ |  |  |  | $A$ |
| B (costs \$200) | В $\checkmark$ | B $\downarrow$ |  |  |  | B |
| C (costs \$200) | C $\downarrow$ | C $\checkmark$ |  | C $\square$ | C | C |
| D (costs \$200) | D $\checkmark$ | D $\downarrow$ | D $\checkmark$ | $\mathrm{D} \square$ | D | D |
| E (costs \$200) | E $\downarrow$ | $\mathrm{E} \quad \checkmark$ |  |  | E | E |
| $F$ (costs \$200) | $F \square$ |  |  | $F \square$ |  | $F \square$ |
| G (costs \$200) | G | G | $\mathrm{G} \square$ | $\mathrm{G} \quad \checkmark$ |  | $\mathrm{G} \square$ |
| H (costs \$200) |  |  |  |  | H $\square$ |  |
|  | 20 voters: | 26 voters: | 11 voters: | 9 voters: | 20 voters: | 14 voters: |
|  | 3.64 | 3.64 |  |  |  |  |
| \$10 | 3.33 | 3.33 |  | 3.64 |  |  |
| $\downarrow$ | 3.03 | 3.03 | 3.03 | 3.03 |  | 3.33 |

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| G (costs \$200) | G | G | $\mathrm{G} \sqrt{ }$ | $\mathrm{G} \checkmark$ |  | $\mathrm{G} \quad \checkmark$ |
| H (costs \$200) |  |  |  |  | $\mathrm{H} \square$ |  |
|  | 20 voters: | 26 voters: | 11 voters: | 9 voters: | 20 voters: | 14 voters: |
|  | 3.64 | 3.64 |  | 3.33 |  |  |
| \$10 | 3.33 | 3.33 | 6.97 | 3.64 |  | 6.67 |
|  | 3.03 | 3.03 | 3.03 | 3.03 |  | 3.33 |

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| A (costs \$200) | $A \square$ | $A \square$ |  | A |  | $A$ |
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| C (costs \$200) | C $\checkmark$ | C $\checkmark$ |  | C $\checkmark$ | C |  |
| D (costs \$200) | D $\checkmark$ | D $\checkmark$ | $\mathrm{D} \square$ | D $\checkmark$ | D |  |
| E (costs \$200) | $\mathrm{E} \quad \checkmark$ | E $\checkmark$ |  |  |  |  |
| F (costs \$200) | $F \square$ |  |  | $F \square$ |  | $F \square$ |
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| H (costs \$200) |  |  |  |  | H $\checkmark$ |  |
|  | 20 voters: | 26 voters: | 11 voters: | 9 voters: | 20 voters: | 14 voters: |
|  | 3.64 | 3.64 |  | 3.33 |  |  |
| \$10 | 3.33 | 3.33 | 6.97 | 3.64 | 10 | 6.67 |
|  | 3.03 | 3.03 | 3.03 | 3.03 |  | 3.33 |

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Theorem: For approval ballots, when all costs are equal the method of equal shares satisfies extended justified representation.

## Ideally it should work for cardinal utilities

| voter |  |
| :---: | :---: |
| 4 ¢ | $170 €$ |
| $2{ }^{2}$ | $25 €$ |
| 9 目 | $124 €$ |
| 7 リール | $93 €$ |
| 2 g | $74 €$ |
| 1 \％${ }^{\text {d }}$ | 155 € |
| 3 m | $130 €$ |

## Extended justified representation（EJR）：

We say that a group of voters $S$ is $T$－cohesive for $T \subseteq C$ if

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A rule $\mathscr{R}$ satisfies extended justified representation if for each election instance $E$ and each $T$－cohesive group $S$ of voters there exists a voter $i \in S$ such that

$$
|A(i) \cap \mathscr{R}(E)| \geq|T|
$$

$u_{i}(c):$ a utility that voter $i$ assigns to $c$ ．

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| voter |  |
| :---: | :---: |
| 45 | $170 €$ |
| 2 \％ | $25 €$ |
| 9 9 自 | $124 €$ |
| 7 リール | $93 €$ |
| 28 | $74 €$ |
| ¢ | $155 €$ |
| 3 m | $130 €$ |

## Extended justified representation（EJR）：

We say that a group of voters $S$ is $(\alpha, T)$－cohesive for $\alpha: C \rightarrow \mathbb{R}$ and $T \subseteq C$ if：

$$
\frac{\operatorname{cost}(T)}{|S|} \leq \frac{b}{n} \text { and } u_{i}(c) \geq \alpha(c) \text { for all } i \in S, c \in T
$$

A rule $\mathscr{R}$ satisfies extended justified representation if for each election instance $E$ and each（ $\alpha, T$ ）－cohesive group $S$ of voters there exists a voter $i \in S$ such that

$$
\sum_{c \in \mathscr{R}(E)} u_{i}(c) \geq \sum_{c \in T} \alpha(c)
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| $3{ }^{3}$ | $130 €$ |

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$$

A rule $\mathscr{R}$ satisfies extended justified representation if for each election instance $E$ and each ( $\alpha, T$ )-cohesive group $S$ of voters there exists a voter $i \in S$ such that

$$
\sum_{c \in \mathscr{R}(E)} u_{i}(c) \geq \sum_{c \in T} \alpha(c)
$$

A rule $\mathscr{R}$ satisfies extended justified representation up-to-one if for each election instance $E$ and each ( $\alpha, T$ )-cohesive group $S$ of voters there exists a voter $i \in S$ and a candidate $d \in C$ such that

$$
u_{i}(d)+\sum_{c \in \mathscr{R}(E)} u_{i}(c) \geq \sum_{c \in T} \alpha(c)
$$

## MES for Cardinal Utilities

1. Each voter is initially given an equal fraction of the budget, i.e., $b / n$ dollars.
G. Pierczyński, P. Skowron, and D. Peters. Proportional participatory budgeting with additive utilities. NeurIPS-2021.

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1. Each voter is initially given an equal fraction of the budget, i.e., $b / n$ dollars.
2. We start with an empty outcome $W=\varnothing$ and sequentially add candidates to $W$.
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To add a candidate $c$ to $W$, we will need that $\sum_{i \in N} p_{i}(c)=\operatorname{cost}(c)$.
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To add a candidate $c$ to $W$, we will need that $\sum_{i \in N} p_{i}(c)=\operatorname{cost}(c)$.
2. For $\rho>0$, we say that a candidate $c \notin W$ is $\rho$-affordable if

$$
\sum_{i \in N} \min \left(\frac{b}{n}-\sum_{c \in W} p_{i}(c), u_{i}(c) \cdot \rho\right)=\operatorname{cost}(c)
$$

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$$
\sum_{i \in N} \min \left(\frac{b}{n}-\sum_{c \in W} p_{i}(c), u_{i}(c) \cdot \rho\right)=\operatorname{cost}(c)
$$

3. If no candidate is $\rho$-affordable for any $\rho$, the rule returns $W$.
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G. Pierczyński, P. Skowron, and D. Peters. Proportional participatory budgeting with additive utilities. NeurIPS-2021.

## MES for Cardinal Utilities

| $b=\$ 2500$ | (65 votes) | (35 votes) | (35 votes) | (50 votes) | (10 votes) | (55 votes) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A (costs \$ 120 ) |  |  |  | A 1 | 2 |  |
| B (costs \$200) | B 30 | B 30 |  |  | B | B |
| C (costs \$500) |  | C 30 |  |  | C | C 10 |
| D (costs \$600) |  |  |  | D 100 | D | D |
| E (costs \$500) | E 10 |  | E 30 |  | E | E |
| F (costs \$180) |  |  | F 10 | F 10 |  | F 10 |
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65 voters:
35 voters:
35 voters


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Theorem: Method of equal shares satisfies extended justified representation up-to-one.

## Can we get EJR (without up-to-one)?

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Theorem: There exists no polynomial-time algorithm that satisfies EJR.
Proof: For one voter this is simply the knapsack problem which is NP-hard.

## Knapsack problem:

We are given a set of items, each with a weight and a value, and two integers: $B, K$. Determine whether there exists a subset of items with total weight not exceeding $B$ and with the total value at least equal to $K$.

## How to use MES with approval ballots?

Given approval ballots we need to decide what is the utility?

There are two main choices:

1. The utility of a voter is the total amount of money spent on approved projects:

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u_{i}(c)=\operatorname{cost}(c) \text { if } i \text { approves } c \text {, and } u_{i}(c)=0, \text { otherwise. }
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2. The utility of a voter is the number of approved projects:

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## Greedy Algorithm:

Select candidates with the highest ratio of value to the weight.

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Such project maximises the value divided by the cost, where the value is the sum of utilities that the voters enjoy from the project, assuming the utility is defined using approach 1.

## Summary



## Motivation [edit]

The method is an alternative to knapsack algorithm which is used by most cities even though it is a disproportional method. For example, if $51 \%$ of the population support 10 red projects and $49 \%$ support 10 blue projects, and the money suffices only for 10 projects, the knapsack budgeting will choose the 10 red supported by the $51 \%$, and ignore the $49 \%$ altogether. ${ }^{[4]}$ In contrast, the method of equal shares would pick 5 blue and 5 red projects.

The method guarantees proportional representation: it satisfies the strongest known variant of the justified representation axiom that is known to be satisfiable in participatory budgeting

## Intuitive explanation [edit]

In the context of participatory budgeting the method assumes that the municipal budget is initially evenly distributed among the voters. Each time a project is selected its cost is


## Analysing data



## Analysing data



## Analysing data



## Analysing data



## Analysing data

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W．Dratt：Method of Equal Shares－Wikipedia


