Barely alternating real almost chains and extension operators for compact lines

Maciej Korpalski

joint work with Antonio Avilés, [1]

Consider an almost chain $\mathcal{A} = \{A_x \subset \omega : x \in X\}$ for some separable linearly ordered set X, so a family of subsets of ω such that for all $x < y, x, y \in X$ the set $A_x \setminus A_y$ is finite. Such a chain is barely alternating if for all $n \in \omega$ we cannot find elements $x_1 < x_2 < x_3 < x_4$ in X satisfying $n \in A_{x_1}, A_{x_3}$, $n \notin A_{x_2}, A_{x_4}$.

We will show that under $MA(\kappa)$, if $|X| \leq \kappa$, then we can straighten our almost chain $\mathcal{A}$ into a barely alternating one by changing at most finitely many elements in each set A_x.

Next we will see how to use this fact to construct extension operator of small norm between spaces of continuous functions on a compact line and its countable discrete extension.

[1] Antonio Avilés and Maciej Korpalski, *Barely alternating real almost chains and extension operators for compact lines*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **118** (2024), Paper No. 148.