

An estimate for the distance from a convex body to subspaces of L_p

For $p \geq 1$, $n \in \mathbb{N}$, and an origin-symmetric convex body K in \mathbb{R}^n , let

$$d_{\text{ovr}}(K, L_p^n) = \inf \left\{ \left(\frac{|D|}{|K|} \right)^{1/n} : K \subseteq D, D \in L_p^n \right\}$$

be the outer volume ratio distance from K to the class L_p^n of the unit balls of n -dimensional subspaces of L_p . We show that there exist absolute constants $c_1, c_2 > 0$ so that

$$c_1 \sqrt{\frac{n}{p}} \leq \sup_K d_{\text{ovr}}(K, L_p^n) \leq c_2 \sqrt{\frac{n+p}{p}}.$$