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Evaluation Report for the PhD thesis of Stanisław Kaźmierowski "Solving Succinct Games"

Evangelos Markakis
Professor, Athens University of Economics and Business

The thesis concerns the algorithmic study of succinct games. These are games that have a succinct representation, in contrast to more general games, where the players' payoff functions cannot be described in an efficient manner. Succinct games form an important family of games, since they often model real decision problems where explicit enumeration is infeasible. On top of that, they also form natural examples of games, where one can attempt to design efficient algorithms for equilibrium computation. For these reasons, succinct games have already attracted significant attention from the relevant research community.

The thesis consists of five main chapters, followed by a final chapter with concluding remarks. The first chapter sets the stage and introduces the topic of interest. It also performs a thorough literature review, which focuses on succinct games that are most related to the games that are studied here. In the subsequent chapters, the thesis focuses on four different variations of succinct games, namely, (i) an extension of Colonel Blotto games with non-bilinear payoff functions, (ii) a class of attack and interception games, where a set of defenders are trying to block an attacker within a network, (iii) one more variation of Colonel Blotto games with flexible tie-breaking rules, (iv) and yet another variant of Colonel Blotto games that incorporate explicit costs associated both with obtaining and deploying resources across battlefields.
I comment first separately for the contributions of each of the chapters 2-5, before providing my overall evaluation.

Chapter 2 – Symmetric Conflicts with Multiple Battlefields: Beyond Bilinear Objectives

In the standard version of Colonel Blotto games, each player has to allocate resources across different battlefields, and each battlefield is won by the player who allocated the most resources. The payoff then to each player is determined by the number of battlefields won, and this leads to bilinear payoff functions. In contrast to this classic version, Chapter 2 investigates games where the payoff can be a more complex function of the individual battlefield results, e.g., threshold-based functions, and this results in non-bilinear payoffs. One example of such games is the majoritarian auction, where a player receives a payoff of 1 only if she wins strictly more than half of the battlefields, and -1 if they lose more than half of them. The contributions of Chapter 2 are as follows:

- First, it is shown that finding the best response to a given strategy of the opponent is NP-complete. However, this does not necessarily mean that finding a Nash equilibrium is an intractable problem.

- The thesis then shows that for computing a Nash equilibrium, it is enough to focus on symmetrized strategies. This helps in reducing the strategy space.
- The next step is then an algorithm that computes payoffs in the symmetrized model. This is referred to as the Clash Matrix algorithm, which is shown to run in polynomial time.
- Finally, based on the above, the thesis is making use of the Double Oracle algorithm, in order to compute a Nash equilibrium. This final part is evaluated experimentally, showing that the algorithm, combined with a heuristic for oracle selection achieves significant speedups.

Chapter 2 demonstrates that while non-bilinear objectives introduce theoretical intractability, a combination of symmetrization and recursive payoff computation makes finding equilibria feasible in practice for these complex strategic environments.

Chapter 3 – Attack and Interception Games in Networks

Chapter 3 explores the computational complexity and algorithmic solutions for a class of multi-player games where an attacker and several defenders interact over a network. This model extends the classic Colonel Blotto framework by introducing network-defined strategy spaces. In the underlying games, there is an attacker, trying to attack a target node within the network. Hence, the attacker selects a path whose endpoint is the targeted node. On the other hand, the defenders are trying to allocate resources to nodes to intercept the attack. This is modeled by choosing an interception probability, that comes at a cost, given by a cost function c(x), where x belongs to [0, 1]. The attacker's strategy space can be exponential in the number of nodes, making the game a natural example of a succinct game.

The chapter starts by first disproving an informal claim that had been made in the paper of Bloch et al, that introduced the model. It then focuses on exploiting the network structure of these games in order to understand better the properties of Nash equilibria. A first important notion in doing so is that of *linkers*, which are essentially nodes that are never targeted at a Nash equilibrium. This allows for a network reduction after removing all the linkers. By digging deeper into the properties of Nash equilibria, the chapter eventually presents an algorithm that finds an equilibrium with a running time that is dependent on the complexity of solving a particular equation (namely, Equation 3.19 within Chapter 3) that determines equilibrium payoffs. As a corollary of this analysis, it is shown that when the defenders have quadratic cost functions, the algorithm runs in time $O(n^3)$. For the more general case of strictly convex cost functions that this chapter is focusing on, the complexity is dependent on the form of the cost functions.

Overall, even though the analysis in this chapter is dependent on certain assumptions, I still find the results to be an important progress. It is true that the complexity of equilibria for the general case is still unknown, and the thesis manages to show how to exploit the network structure, so as to come up with an algorithm that works efficiently for quadratic cost functions.

Chapter 4 – Equilibria of the Colonel Blotto Game with Flexible Tie-breaking

This chapter studies another variation of Colonel Blotto games, where tie-breaking can create either costly inefficiency or spill-over benefits. In the classic version of Colonel Blotto, each

player receives half a unit of payoff from battlefields where the players have placed an equal amount of resources. The model presented in this chapter allows for this payoff to be controlled by a parameter α. The objective is to understand how equilibrium behavior changes when the strategic value of a tie is allowed to vary, and to characterize the resulting equilibrium set. In doing so, the analysis highlights both the robustness and the limits of known equilibrium constructions under different assumptions about the efficiency of ties.

One of the main contributions of the chapter is to highlight that by allowing this more flexible tie-breaking rules, the resulting games may have many equilibria, under certain natural and rather reasonable assumptions. This shows that in general, Nash equilibria is not a good prediction model for the behavior of the players in these games. The thesis then proceeds to provide a more detailed understanding of the equilibrium set. Among others, it establishes results on pure strategies that are never used in any equilibrium, and it also establishes that as long as α is relatively small, then these games do not have weakly dominated strategies (and hence one cannot hope for a further refinement of the equilibrium set). Finally, the chapter concludes with discussing the existence of pure equilibria as well as the existence of asymmetric equilibria.

Chapter 5 – Equilibria of the Colonel Blotto Game with Costs

This chapter examines yet another variation of Colonel Blotto games. Namely, it introduces a model that incorporates explicit costs for both obtaining and deploying resources. This formulation captures the strategic reality that resources are often not free and that the cost of acquisition can influence allocation decisions. The thesis also contains motivating examples to justify the model under consideration.

The most significant finding in this chapter is that every instance of the Colonel Blotto game with costs is strategically equivalent to a standard Colonel Blotto game with one additional battlefield. This transformation is crucial because it allows the cost-augmented game to inherit the well-understood properties of the standard games, such as the interchangeability of Nash equilibria (where any pair of optimal strategies chosen independently by the players still forms an equilibrium). Most importantly, it also allows to find Nash equilibria by using linear programming. Although the latter does not always lead to an efficient algorithm, since the size of the linear program depends on the number of the available resources, it allows for efficient computation when this parameter is small. Finally, the chapter also provides an experimental evaluation of the algorithm, regarding its running time and the number of resources obtained at equilibrium.

Overall evaluation of the thesis

To summarize my final conclusions, I believe that the thesis

- introduces or studies further new variants of the original Colonel Blotto game, which are well motivated with illustrating examples.
- provides a coherent treatment of succinct games centered around these variations.

- for several classes of these games, it provides polynomial time algorithms, such as the games with quadratic cost functions in Chapter 3 or games with a small number of available resources in Chapter 5.
- provides valuable insights into the equilibrium structure in most of the chapters. This includes the exploitation of the network structure in Chapter 2, the effect of the tie-breaking rules in Chapter 4, and the strategic equivalence to zero-sum games in Chapter 5.
- complements the theoretical analysis with some experimental evaluations so as to derive further conclusions on the performance of the proposed algorithms.

Based on the above, I find the thesis to satisfy the PhD standards and consider it sufficient to grant a PhD degree. I would only like to suggest a few corrections, which I consider to be only a minor revision, as follows.

Minor comments and suggested edits.

1. In Chapter 3, it seems that in a pure strategy profile, the defenders that are not targets have an incentive to just play $x_j = 0$ for every such j, according to Equation 3.1, where their utility function is defined. Only the targeted node may have an incentive to not play 0. This is a property that should be stated.
2. In Chapter 3, is c(0) = 0? This seems to be the case but I did not see it to be explicitly stated.
3. Typo in Page 36: In the first paragraph of the proof of Example 3.2, the last sentence should be “The payoff function to 1 *is*”.
4. In Chapter 4, it may look initially that there is a possible conflict between Proposition 4.6 and 4.7. It should be explicitly clarified that there is no contradiction here, and that a pure strategy cannot satisfy both the premises of Proposition 4.6 and of Proposition 4.7. And this is because if one uses less than n* battlefields, then it cannot be the case that it allocates at most 2D/n resources in each battlefield and still manages to allocate all D resources.
5. Typo in Page 69: *illustrate* in the sentence right before Proposition 4.12.
6. In Chapter 5, it should be mentioned that the algorithm is a pseudopolynomial time algorithm, given the dependence on the number of resources.
7. References: There is a repetition of the Behmezad et al reference (of the SODA paper).

Please feel free to contact me for any further information regarding my assessment of the thesis.

Sincerely,

Evangelos Markakis

EVANGELOS MARKAKIS
05/03/2026 00:27