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**Reviewer report the thesis "SOLVING SUCCINCT GAMES" by Stanisław Kaźmierowski**

## Summary of the Thesis

The thesis investigates the algorithmic and structural properties of succinctly represented games, with a particular focus on conflicts involving multiple battlefields and network-based attack–defense interactions. The unifying theme is the tension between compact game representations and the computational complexity of equilibrium analysis. While succinctness allows for realistic modeling of large strategic environments, it often introduces significant algorithmic challenges, especially when traditional normal-form representations become infeasible.

After a broad and well-structured introduction to succinct games and equilibrium computation, the thesis develops this perspective through a sequence of four case studies, each corresponding to a research paper by the author (sometimes in collaboration). These chapters cover (i) symmetric multi-battlefield conflicts with non-bilinear objectives, (ii) attack and interception games on networks, (iii) Colonel Blotto games with flexible tie-breaking rules, and (iv) Colonel Blotto games with explicit costs.

A recurring methodological contribution of the thesis is the careful exploitation of structural properties—such as symmetry, monotonicity, or network structure—to overcome the apparent intractability of large strategy spaces. In particular, the thesis combines complexity-theoretic results that delineate the limits of tractability with constructive algorithms that are efficient in practice. This approach allows the author to show that, despite worst-case hardness results, meaningful subclasses of succinct games admit polynomial-time equilibrium computation.

Overall, the thesis provides a coherent and well-motivated exploration of the "algorithmic frontier" of succinct games, offering both negative results (hardness barriers) and positive ones (efficient algorithms and structural insights), and it situates these contributions very well within the existing literature.

## General Assessment and Strengths

The thesis is very well written and clearly structured. Each chapter is carefully motivated, and the progression from general background to specific contributions is smooth and convincing. Although the games studied across the chapters may appear similar at first glance—often involving multiple battlefields or network structures—the thesis convincingly demonstrates that their use cases, modeling assumptions, and algorithmic challenges differ in essential ways. This diversity is, in my opinion, one of the strengths of the dissertation.

Most of the proofs appear correct, and the technical level is appropriate for a PhD thesis in algorithmic game theory. I particularly appreciated the introduction of Clash Matrices and the associated dynamic programming approach (Section 2.5), which I find elegant, conceptually clean, and potentially useful beyond the specific application considered in the thesis.

From a presentation perspective, the number of typos is relatively small. There are, however, recurring minor grammatical issues, such as the incorrect use of a/an (for example, "a NE" instead of "an NE"), which could be corrected easily but do not detract from the overall quality of the work.

## Specific Remarks and Minor Concerns

I have some doubts regarding the proof of Lemma 3.8. As it is currently presented, it is not entirely clear to me that the argument holds in full generality. That said, this issue does not affect the main results of the chapter, since the

crucial consequence is Corollary 3.9, which I believe to be correct and sufficient for the subsequent developments. I would therefore view this point as a minor technical concern rather than a substantive flaw.

**Questions and Suggestions (Non-blocking) [in case there should be some]**

- On complexity statements (Section 2.7): The thesis states that a certain task "cannot be achieved in polynomial time unless P = NP." Could the author comment on the motivation for phrasing the result in this way rather than stating directly that the problem is NP-hard? While the two formulations are closely related, it would be interesting to understand whether there is a conceptual or technical advantage to the chosen wording in this context.
- On neutral nodes and linkers: The thesis introduces the notion of neutral nodes (linkers) and provides Proposition 3.4, which gives a sufficient condition: every linker is a neutral node. Have you considered whether a converse characterization is possible? In particular, is there a simple condition under which a neutral node must be a linker, or is there an easy counterexample showing that no such characterization can hold? A brief discussion could further clarify the conceptual landscape here.
- On the algorithmic setup in Chapter 5: Regarding the experimental setup and algorithmic choices, I wonder whether it might sometimes be preferable to solve several smaller linear programs (for fixed, relatively small values of D^\max) instead of a single large LP. In practice, large LPs can become computationally heavy, whereas one is often "lucky" and obtains the desired solution quickly from a smaller instance. While this does not affect the correctness of the results, it could be an interesting practical consideration or alternative heuristic to mention.

**Overall Evaluation**

All in all, this is an excellent thesis that addresses a well-chosen and timely topic. It makes several original and technically sound contributions and demonstrates a strong command of both the theoretical foundations and the algorithmic aspects of modern game theory. The results are clearly presented, well-motivated, and carefully situated within the existing literature.

I particularly enjoyed reading the thesis. Although I was already familiar with Colonel Blotto games, the work offered new insights and perspectives that deepened my understanding of the area, which is a strong indication of its quality and originality.

I strongly believe that the thesis should be accepted for defense. If a grade is to be assigned, I am confident that it deserves the highest mark (A – excellent).

Dušan Knop