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TO WHOM IT MAY CONCERN:

I am happy to share some remarks on the scientific contributions of **Stanisław Cichomski** presented in his Ph.D. thesis. Let me start by expressing my unconditional recommendation that Cichomski be awarded a Ph.D. degree in mathematics (doktor w dziedzinie nauk ścisłych i przyrodniczych, w dyscyplinie matematyka).

The thesis is based on five published papers and one paper accepted for publication. They are all focused on coherent distributions.

A collection of probabilistic beliefs held by a group of agents is called coherent if it can be represented by a single probability space and the probabilistic beliefs of every agent can be obtained by conditioning on a (typically different) σ-field. This mathematical concept is a model for human decision makers who are rational and have an identical opinion about the general structure of reality but have access to different sources of information.

There is a body of literature on related topics, some of them inspired by modern uses of technology. Consider several experts represented by sub-σ-fields who are all trying to predict the probability of a common event. A natural question is if there is a way to combine their predictions to come up with a better forecast. Introduced this way in the mid 80's onwards, such combinations typically take the form of weighted averages. The field has found a renewed interest in the current age of social networks. In particular, some authors recommend both linear and nonlinear combinations, others develop a mathematical framework to combine predictions when experts use "partially overlapping information sources." Others use the model for the case of two experts in prediction markets who take turn in updating their beliefs. There are also applications to economics, banking and finance, meteorology, and analytic philosophy.

Coherent distributions can be represented as random variables taking values in $[0, 1]$.

Chapter 2 of the thesis is devoted to estimates of the maximum possible difference between coherent distributions in the L^p norm. The approach involves identifying extreme coherent

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distributions, an interesting and significant challenge in its own right.

The last topic mentioned above is developed in Chapter 3. Cichomski asks a seemingly easy question whether there exist non-atomic extreme coherent distributions. Then he proves that indeed they exist.

Chapter 4 is devoted to the case when two experts have access to independent sources of information. Cichomski proved a conjecture that had been stated in my paper with Pitman. The conjecture is a sharp bound on the discrepancy of opinions.

Chapter 5 takes us to a new subfield, namely, the study of discrepancy between opinions of n experts, with $n \geq 2$. The main result of Chapter 5 is a striking theorem on the maximal discrepancy in the L^1 norm. What I find personally fascinating is that Cichomski's explicit formulas have different form for $n = 2, 3, 4$, and starting with $n = 5$, they have the same form for all $n \geq 5$.

Chapter 6 contains a generalization of my result with Pal on the discrepancy between coherent opinions in L_∞ norm. The result is generalized from 2 to many agents, and a new, better, proof is supplied.

The seventh chapter contains a generalization of the classical Doob's inequality from the martingale context to the context of random variables obtained by conditioning on partly nested filtrations. Cichomski's inequality is supplied with a sharp constant, a feature greatly appreciated in this area of research.

Cichomski's methods include a wide variety of mathematical tools, such as combinatorics, symmetrization, reduction, graph theory, dynamical systems, dynamic programming, optimization, martingale theory and associated inequalities.

Cichomski's strengths show up in multiple ways. Some proofs are long and complicated sequences of steps, for example, reductions. But the most significant contributions are new ideas, such as studying extremal coherent distributions, or finding connections with other fields of mathematics.

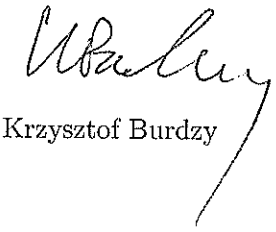
The articles comprising the thesis are the most extensive contribution to the literature on coherent distributions coming from a single author (more precisely, from a pair of co-authors).

The thesis is written in an accessible and clear way. Multiple illustrations are very helpful.

The quality of the thesis is at the level of the top doctoral theses at my university (University of Washington).

I am truly impressed by the high quality and significance of research presented in the Ph.D. thesis of Stanisław Cichomski. I believe that it should be nominated for an award (wyróżnienie rozprawy doktorskiej).

Sincerely yours,



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