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Report on the doctoral dissertation:

Mathematical analysis of obstacle approximation strategies for incompressible flow

The disertation is devoted to the problem of flow in moving frame. Sadokat Malikova in Chapter 1 gave an introduction to the problem and explaination of the physical motivation of the problem. The viscosity penalization method is a tool used e.g in the case of the motion of the flow around moving rigid body. Such method was introduced in the work of Hoffmann and Starovoitov and San Martin, Starovoitov, Tucsnak. Such concept is called "solidification" and was used to show the existence of the weak solution of the problem. Another interesting point is to achieve the higher regularity of the approximate solution. The problematic issue is the jump function in the diffusion term. Tool here is to define the tangential regularity. Such method was introduced by Bony and later on developed by Chemin, Danchin and collaborators.

The volume penalization method called the Brinkman penalization can be found e.g. in the work of Angot or Bruneau and Fabrie, etc.

Chapter 2 contains the first result of the thesis, which was published in [23] and it described the viscosity penalized stationary Navier-Stokes system in bounded domain in 2D case. The homogeneous Dirichlet boundary condition is consider on the boundary of domain. The kinematic viscosity $\nu(x)$ is a discontinuous function,

$$\nu_m(x) = \{ \begin{array}{l} 1,\, x \in \Omega \setminus \Omega_S, \\ m,\, x \in \Omega_S. \end{array} \tag{1}$$

where the Ω_s denotes the domain occupied by the rigid obstacle.

First result of the Chapter 2 is the existence of a weak solution which is obtained by the standard Galerkin method and using compactness arguments. The main result of this chapter is the L^∞ bound for the velocity gradient. Due to discontinuity, the tool to improve the regularity is the tangential regularity approach.The first part of the proof is the deriving tangential regularity for the approximate Stokes system. In the second part the approximate Navier-Stokes system is investigated. Very important part is the application of tangential regularity on $div\partial_x U_m$, which is no longer zero. It implies that the pressure term has to be included in the weak formulation and we have to apply the Bogovskii type estimate. After that the regularity in the normal direction is studied. Since the normal derivative of the jump function is not defined, a model scenario has to be introduced. In this model, the interior of the obstacle domain Ω_s is introduced as the half-space where $x_2 < 0$, while the exterior corresponds to the region where $x_2 > 0$,

$$-\mathrm{div}[\nu_m(x) Du_m] + \nabla p = f \text{ in } R^2,$$

where

$$\nu_m(x_2) = \{ \begin{array}{l} 1,\, x_2 > 0, \\ m,\, x_2 < 0, \end{array} \tag{2}$$

and the tangential vector field have a form $\partial_X = \partial_{x_1}$. By the splitting the problem is reduced to one dimension.Moreover, the problem is reformulated in curvilinear coordinates and the system is extended to R^2. Finally $\nabla u_m \in L^\infty(\Omega)$. The numerical simulation is also presented.

Chapter 3 focused on paper [17] where the stationary Navier-Stokes system is considered in 2D and 3D case around the obstacle and penalization is used of the Brinkman type. The rate of convergence is investigated. Moreover, also the viscosity penalization is applied and again the convergence is studied. Moreoverm combination of both penalizations and the convergence rate is again studied. Finally, also in this chapter the numerical simulation is presented.

Chapter 4 deals with the approximation of time dependent system using the mixed penalty approximation. The existence of weak solutions is shown using auxiliary method introduced in Hofmann, Starovoitov. Moreover, the regularity of the rigid velocity is shown.

Summarizing the report I am fully convinced that a written dissertation thesis is of very good level. The results presented are original and show that has mastered several techniques and methods both in functional analysis and in PDE theory with applications for fluid mechanics. In my opinion, this is a nice doctoral thesis. I fully recommend it for the defense in front of the respective committee.

Sincerely yours,

Šárka Nečasová