E. Wagner
Université Paris Cité
IMJ-PRG – UMR 7586
Bâtiment Sophie Germain 8 Place Aurélie Nemours
Boite courrier 7012 75205 PARIS Cedex 13
email : emmanuel.wagner@imj-prg.fr

Paris, 14th of November, 2025

Second Report on the thesis of Ramazan Yozgyur entitled
" Khovanov-Rozansky sl_n-homology for periodic links".

The research area of Ramazan Yozgyur is called quantum topology. This subject is born after the discovery of Jones of his famous polynomial link invariant. It developped rapidly after through the works of Reshetikhin-Turaev, Melvin-Kirby, Turaev-Viro and many others. At the interface of low dimensional topology, representation theory (of quantum groups) and mathematical physics, it is based at the same time on combinatorial descriptions of topological objects (Reidemeister theorem for knots, links and tangles, Lickorish-Wallace and Kirby theorem for surgery presentations of 3-manifolds, triangulations, Heegaard splitings and Singer's theorem,...) and on the work of Drinfled, Jimbo and others on quantum groups (and their representation theory), integral systems and mathematical physics. It fits into the topological quantum fiel d theory as axiomatize by Atiyah.

Since the earlys days, the question of the topological and geometrical meaning of the quantum invariants of links was a central one. The subject knew a huge renewal with the categorification program, initiated with the categorification of the Jones polynomial, by Khovanov. This lead to the theory of link homologies and provided a new angle on the question of the topological and geometrical meaning of the (categorified) quantum invariants of links.

This thesis deals with one of this link homology theory namely the Khovanov-Rozansky sl_n link homology and with one specific question namely understanding how certain symetries of some links reflects in them.

More precisely the thesis considers periodic links, namely links who admits a diagram invariant under a rotation of a given angle $2\pi/m$ of the plane. Examples are closures of braids of the form β^m for some given braid β and some integer m. Hence the group $\mathbb{Z}/m\mathbb{Z}$ acts on the diagram (the subgroup generated by the rotation). Perodic links are also just links and one can consider any associated invariant and the thesis studies the Khovanov-Rozansky sl_n link

homology which is computed from a cochain complex $[[D]]$ of graded $Sym[X_1, \ldots, X_n]$-modules associated to any diagram D.

The main results of the thesis are the followings :

1. For a given symetric link diagram D, the action of $\mathbb{Z}/m\mathbb{Z}$ descends to an action on $[[D]]$
2. If D and D' are symetric diagrams related by a symetric Reidemeister move, then $[[D]]$ and $[[D']]$ are homotopy equivalent equivariantly with respect to the action of $\mathbb{Z}/m\mathbb{Z}$.
3. A generalization of the Borodzik-Politarczyk criterion on Khovanov homology to Khovanov-Rozansky sl_n link homology.

The thesis is divided as follows. The first two sections are devoted to a very quick introduction and an overview of the necessary preliminaries on links, quantum link invariants, homological algebra and link homologies. The third section reviews, how symetries reflects on some invariants, including on Khovanov homology, a result by Borodzik and Politarczyk. The fourth section reviews the background on webs, foams and a definition of sl_n-link homologies over $Sym[X_1, \ldots, X_n]$.The fifth and the sixth are the main part where one provide statement and proof for the first two main results mentionned earlier. The fifth section also includes classical considerations around specialization of the variable's X_i's to fixed complex values. The seventh section discusses spectral sequence arising from a splitting of the hypercube ; such argument appeared earlier on the work of Turner but here the equivariant setting is incomporated. The last section establishes the last main result mentionned earlier.

The main achievement of the thesis is to make everything work over a field of any characteristic, and in particular many of the technical issues come from signs.

Overall this thesis answers nicely a very natural question, generalizing the work of Chbili and Borodzic-Politarczyk using the foam version of the Khovanov-Rozansky sl_n link homology.

The manuscript has been edited according to the first reports and the exposition has been definitively improved.

I definitively deem the thesis acceptable for a PhD and strongly recommend its defense.

E. Wagner, Professeur des universités
Université Paris Cité, Paris

Minor remarks.

— Add a reference to the early work of Chbili on Khovanov homology and symmetries.

UNIVERSITE PARIS CITÉ
IMJ-PRG
CNRS UMR 7586

E. Wagner
Université Paris Cité
IMJ-PRG – UMR 7586
Bâtiment Sophie Germain 8 Place Aurélie Nemours
Boite courrier 7012 75205 PARIS Cedex 13
email : emmanuel.wagner@imj-prg.fr

Paris, 28 février, 2025

Report on the thesis of Ramazan Yozgyur entitled
" Khovanov-Rozansky sl_n-homology for periodic links".

The research area of Ramazan Yozgyur is called quantum topology. This subject is born after the discovery of Jones of his famous polynomial link invariant. It developped rapidly after through the works of Reshetikhin-Turaev, Melvin-Kirby, Turaev-Viro and many others. At the interface of low dimensional topology, representation theory (of quantum groups) and mathematical physics, it is based at the same time on combinatorial descriptions of topological objects (Reidemeister theorem for knots, links and tangles, Lickorish-Wallace and Kirby theorem for surgery presentations of 3-manifolds, triangulations, Heegaard splitings and Singer's theorem,...) and on the work of Drinfled, Jimbo and others on quantum groups (and their representation theory), integral systems and mathematical physics. It fits into the topological quantum fiel d theory as axiomatize by Atiyah.

Since the earlys days, the question of the topological and geometrical meaning of the quantum invariants of links was a central one. The subject knew a huge renewal with the categorification program, initiated with the categorification of the Jones polynomial, by Khovanov. This lead to the theory of link homologies and provided a new angle on the question of the topological and geometrical meaning of the (categorified) quantum invariants of links.

This thesis deals with one of this link homology theory namely the Khovanov-Rozansky sl_n link homology and with one specific question namely understanding how certain symetries of some links reflects in them.

More precisely the thesis considers periodic links, namely links who admits a diagram invariant under a rotation of a given angle $2\pi/m$ of the plane. Examples are closures of braids of the form β^m for some given braid β and some integer m. Hence the group $\mathbb{Z}/m\mathbb{Z}$ acts on the diagram (the subgroup generated by the rotation). Perodic links are also just links and one can consider any associated invariant and the thesis studies the Khovanov-Rozansky sl_n link

homology which is computed from a cochain complex $[[D]]$ of graded $Sym[X_1, \ldots, X_n]$-modules associated to any diagram D.

The main results of the thesis are the followings :

1. For a given symetric link diagram D, the action of $\mathbb{Z}/m\mathbb{Z}$ descends to an action on $[[D]]$
2. If D and D' are symetric diagrams related by a symetric Reidemeister move, then $[[D]]$ and $[[D']]$ are homotopy equivalent equivariantly with respect to the action of $\mathbb{Z}/m\mathbb{Z}$.
3. A generalization of the Borodzik-Politarczyk criterion on Khovanov homology to Khovanov-Rozansky sl_n link homology.

The thesis is divided as follows. The first two sections are devoted to a very quick introduction and an overview of the necessary preliminaries on links, quantum link invariants, homological algebra and link homologies. The third section reviews, how symetries reflects on some invariants, including on Khovanov homology, a result by Borodzik and Politarczyk. The fourth section reviews the background on webs, foams and a definition of sl_n-link homologies over $Sym[X_1, \ldots, X_n]$.The fifth and the sixth are the main part where one provide statement and proof for the first two main results mentionned earlier. The fifth section also includes classical considerations around specialization of the variable's X_i's to fixed complex values. The seventh section discusses spectral sequence arising from a splitting of the hypercube ; such argument appeared earlier on the work of Turner but here the equivariant setting is incomporated. The last section establishes the last main result mentionned earlier.

The main achievement of the thesis is to make everything work over a field of any characteristic, and in particular many of the technical issues come from signs.

Overall this thesis answers nicely a very natural question, generalizing the work of Chbili and Borodzic-Politarczyk using the foam version of the Khovanov-Rozansky sl_n link homology.

There are a couple of minor remarks whose details can be found on the next page but they do not affect my recommendation.

I definitively deem the thesis acceptable for a PhD and strongly recommend its defense.

E. Wagner, Professeur des universités
Université Paris Cité, Paris

Minor remarks.

— Add a reference to the early work of Chbili on Khovanov homology and symmetries.
— The quotient link seems nowhere defined.
— Formulation of Theorem 3.8 seems a bit strange : $g_{\overline{K}}$ is either the Seifert genus or a non positive integer that exists.
— In the title of Section 6, it should be Theorem 5.25, not Lemma 5.25.