Durham University

Shaped by the past, creating the future

Department of Mathematical Sciences

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18th September 2025.

**Ramazan Yozgyur Thesis Report.**

# Executive summary and recommendation.

The revised thesis is a somewhat improved document; my recommendation remains unchanged.

This is a thesis concerned with studying links in the 3-sphere $S^3$ which are periodic with respect to a cyclic action on $S^3$. There are invariants of such links that arise from considering the induced action on Khovanov homology. The thesis extends these invariants to the $sl_N$ analogues of Khovanov homology known as Khovanov-Rozansky homology.

The results presented are novel and significant. I enjoyed reading this thesis – I learnt something from it. I am happy to deem that this revised thesis is sufficient to award a doctorate in the natural sciences, in the field of mathematics, to the candidate Ramazan Yozgyur.

# Summary of the thesis.

In this section we describe and comment on the thesis chapters.

## Introduction.

The thesis begins with a short introduction outlining the main contributions and providing forward references.

## Khovanov homology.

There is a chapter giving a brief summary of the construction of Khovanov homology, done in such a way as to prepare the ground for the construction of Khovanov-Rozansky homology. The introduction to knot theory is done in a very informal way: this referee would have appreciated a little more formality and polishing.

## Periodic links.

The thesis gives the rough state-of-play of the study of periodic links, closing with the invariants of Borodzik-Politarczyk derived from Khovanov homology. An illustrative example is worked through at the close of the chapter.

It is these invariants of Borodzik-Politarczyk that the thesis aims to generalize to Khovanov-Rozansky homology.

### Webs, foams, and categories.

The thesis gives the construction of equivariant Khovanov-Rozansky homology via the formalism of webs-and-foams. There are some inaccuracies in this chapter in the rush to get through the definitions, but nothing too serious.

### Specializations.

The next chapter considers specializations of the equivariant Khovanov-Rozansky homology and packages such specializations as spectral sequences. It then goes on to introduce the action of the cyclic group on the equivariant complex and considers the algebraic interpretation of such an action. It might have made sense to move this to its own chapter. The proofs follow expected lines: there is much homological algebra to check combinatorial equivalences.

The main theorem – that a periodic Reidemeister move induces an equivalence of the algebraic structures – is postponed to the following section.

### Proof of main theorem.

This chapter forms much of the work of the thesis, in proving that the invariant constructed is well-behaved under periodic Reidemeister moves. There is some genuine work undertaken in this chapter, and it is this chapter, building on the earlier definitional work, that makes this a PhD thesis.

The proofs, essentially, are direct checks that chain complexes and chain maps are playing well with the cyclic group actions. There are quite a few moving parts of which to keep track.

### The skein spectral sequence.

The relationship of the periodic invariant to the classical invariant takes the form of a spectral sequence, and this is made precise in this section.

### Polynomial invariants.

Finally, one reduces to the Poincaré polynomials of the classical $sl_N$ invariant in order to deduce a periodicity obstruction, generalizing earlier work arising from Khovanov homology.

## Quality of Exposition

The dissertation is well-structured, logically progressing from fundamental concepts to increasingly complex results. The introduction provides a clear motivation for the study, situating the work within the broader context of link homology. Each chapter is well-organized, with generally precise definitions, generally clearly stated theorems, and rigorous proofs.

The author also provides some illustrative examples, which aid in understanding the invariants discussed.

There are occasional inaccuracies and infelicities but nothing which prevents this thesis from being accepted for the award of Doctor of Philosophy.

## Some minor corrections.

Here is a list of some suggested improvements to the thesis, most of which have not been addressed in the revision. These need not be incorporated before the thesis is accepted. My suggestions are not exhaustive, but hopefully indicative of the kind of thing that could be improved.

- Def 2.1 – is a knot or a link a submanifold or an embedding? There is no harm in being clear.
- After Example 2.2 you jump from discussing a link to thinking about a diagram without being precise about the jump.
- Definition 2.5 is not phrased as a definition but as an instruction.
- Definition 2.7 is not precise enough.
- Theorem 2.9 in its statement doesn't mention diagrams.
- In general, the rest of Chapter 2 could do with a read-through with an eye to making statements more precise. This can be done fairly easily without the expense of lengthening the exposition dramatically.
- Example 3.3 should be illustrated.
- Remark 3.4 – is this conjecture still a conjecture?
- Def 4.3 – is it really a *closed* web that you wish to define?
- Def 4.5 is preceded by a line that it should rather incorporate.

Yours faithfully,

A. J. Lobb.

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26th February 2025.

**Ramazan Yozgyur Thesis Report.**

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It is these invariants of Borodzik-Politarczyk that the thesis aims to generalize to Khovanov-Rozansky homology.

### Khovanov-Rozansky homology

The thesis gives the construction of equivariant Khovanov-Rozansky homology via the formalism of webs-and-foams. There are some inaccuracies in this chapter in the rush to get through the definitions, but nothing too serious.

### Specializations.

The next chapter considers specializations of the equivariant Khovanov-Rozansky homology and packages such specializations as spectral sequences. It then goes on to introduce the action of the cyclic group on the equivariant complex and considers the algebraic interpretation of such an action. It might have made sense to move this to its own chapter. The proofs follow expected lines: there is much homological algebra to check combinatorial equivalences.

The main theorem – that a periodic Reidemeister move induces an equivalence of the algebraic structures – is postponed to the following section.

### Invariance.

This chapter forms much of the work of the thesis, in proving that the invariant constructed is well-behaved under periodic Reidemeister moves. There is some genuine work undertaken in this chapter, and it is this chapter, building on the earlier definitional work, that makes this a PhD thesis.

The proofs, essentially, are direct checks that chain complexes and chain maps are playing well with the cyclic group actions. There are quite a few moving parts of which to keep track.

### Spectral sequences.

The relationship of the periodic invariant to the classical invariant takes the form of a spectral sequence, and this is made precise in this section.

### Polynomial invariants.

Finally, one reduces to the Poincaré polynomials of the classical $sl_N$ invariant in order to deduce a periodicity obstruction, generalizing earlier work arising from Khovanov homology.

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## Some minor corrections.

Here is a list of some suggested improvements to the thesis. These need not be incorporated before the thesis is accepted. In general, it is the earlier chapters (I assume these were "filled in" once the main mathematical work of the later chapters was accomplished) that could do with a little attention. My suggestions are not exhaustive, but hopefully indicative of the kind of thing that could be improved.

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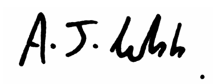

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