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Please find below the report on the Doctoral dissertation ``Efficient Data Structures and Graph Width Parameters", submitted by Marek Sokołowski.

The thesis deals with parameterized complexity and graph theory, with a focus on *structural graph decomposition parameters*. Such parameters have played a significant role in graph theory and for graph algorithms in the past 30 years.

The graph parameters considered in this thesis are treewidth, rankwidth, and twinwidth. Treewidth and rankwidth are classic parameters that have been studied for decades, and are fairly well understood. On the other hand, the twin-width parameter was only defined in 2020, and has received considerable attention since then. Despite the considerable attention, twinwidth is still not well-understood at all. The submitted dissertation, which consists of a comprehensive introduction and literature survey followed by an edited collection of 5 research papers, makes breakthrough contributions to our understanding of treewidth and rankwidth, as well as significant steps forward to the understanding of twin-width.

The dissertation consists of several individual research works, which have been edited to constitute the chapters of the thesis. Together the different chapters form a coherent body of work that are tied together by the introduction: there is a clear focus on the algorithmic aspects of basic graph parameters, and the structural insights behind the algorithms. I will now discuss each of the constituent pieces.

Dynamic Treewidth: The first, and perhaps the most impressive, component of the thesis is the work on dynamic treewidth. The treewidth of a graph is a parameter that measures how close a graph is to being a tree. It is *extremely* well studied, and there is a wealth of algorithms that can be deployed when a (sufficiently good) tree decomposition of the graph is given as input. Many algorithms have been designed to compute the treewidth of an input graph G, the different algorithms offer different trade-offs in terms of their running time and the quality of the output decomposition (a survey of these algorithms is given in the submitted dissertation). However, up until the submitted work, no algorithm that dealt with a dynamically changing graph were able to maintain a decomposition whose width is a function only of the treewidth of the (current) input graph. The work in this dissertation achieves a dynamic data structure with near-optimal width (only a small constant factor off from the true treewidth) and amortized running time per operation that is ``almost poly-logarithmic".

The fact that the dependence on treewidth is so good is important – because this allows for a dynamic variant of the classic Courcelle's theorem, paving the way for multiple new research avenues. First: can the success be repeated, and can we achieve dynamic data structures for other (related) graph parameters and graph problems (this is addressed in the next two chapters of the thesis). And second: can the new dynamic data structures be used to design faster parameterized algorithms? A positive answer to this question came in a recent breakthrough by Korhonen, Pilipczuk and Stamoulis, who gave built on the insights of this work to get a near-linear time algorithm for Minor Containment and the Disjoint Paths problem. In summary – this work is absolutely outstanding: it greatly pushes forward the understanding of a fundamental graph parameter, and has in short time shown to be very useful and influential.

Dynamic Rankwidth:

This is a the obvious follow-up question to the work on Dynamic Treewidth. In essense Rankwidth is ``treewidth for dense graphs", and shares many properties with treewidth. Thus, having a dyncamic algorithm for treewidth in hand it is very natural to ask whether such an algorithm can also be achieved for rankwidth. The second chapter in this thesis achieves this goal. It is very important to note that while the question is an obvious follow up, the answer (the dynamic algorithm for rankwidth) is not just an adaptation of the algorithm for treewidth. There are many substantial challenges that have to be overcome.

Additionally the thesis presents a strengthened the data structure to also allow for ``dense" edge additions that allow many edges to change at the same time. This immediately leads to the first near-linear time algorithm to compute the rankwidth of a graph – that on its own would be considered a major result.

Dynamic Approximation on Planar Graphs:

Having tackled dynamic tree decompositions, it is very natural to ask whether one can get dynamic versions for algorithms that are based on tree decompositions. One such class of algorithms are approximation schemes for various problems on planar graphs. The dissertation gives dynamic approximation schemes for two ``canonical" graph problems: Independent Set and Dominating Set. In this setting there is still significant room for further work: in the static setting the algorithms for the ``canonical" problems readily generalize to a host of other problems. Here, that appears much harder (even if the results do generalize somewhat). Additionally, the results for Dominating Set only work for bounded degree graphs. That said, the results in this chapter are nevertheless impressive and important ``proof of concept" results that show that the results for dynamic treewidth often can be carried over to applications of tree decompositions. I fully expect that the paper that the chapter is built on will be the first of many works in this direction, and, as such, be highly influential.

Compact Representation of Bounded Twin-Width graphs:

Twinwidth is a new graph parameter that vastly generalizes treewidth and rankwidth. The generality of course comes at a price: many problems that are efficiently solvable on graphs of bounded rankwidth are not on graphs of bounded twinwidth. Additionally, at present, we do not know of any good algorithms to compute (or approximate) the twin-width of a graph. Thus, most work (including the two chapters on twin-width in this thesis) deal with the case where a contraction sequence (or equivalent witness of twin-width being small) is provided as input.

The first treatment of twin-width in this dissertation follows the general theme of data-structure related questions: one asks whether an input graph G of bounded twin-width can be stored using very little space (linear in the number n of vertices), such that one can efficiently answer adjacency queries (given two vertices, report whether they are adjacent or not). This work presents such a scheme, which compares favorably with previous related work.

Bounded Twin-Width graphs are Quasi-Polynomially Chi-Bounded:

It is well-known that, in general, the clique size and the chromatic number of a graph do not need to be related (the chromatic number is always at least the clique size, but there are graphs with no clique of size 3 and arbitrarily large chromatic number).

It is a very active area of research in structural graph theory to identify the classes of graphs where the chromatic number can be bounded in terms of the size of the maximal clique, and trying to make this bound as tight as possible.

It was known that in graphs of bounded twin-width, the chromatic number is bounded exponentially in the size of the maximum clique. It was posed as an open problem (in 2021) whether the bound on the chromatic number should be polynomial instead. The final chapter in this thesis provides a partial resolution of this open problem: it contains a quasi-polynomial bound on the chromatic number. It is quite pleasing to see that following the work in this dissertation (and building on the methods developed here), Bourneuf and Thomasse obtained a polynomial bound.

Summary Evaluation:

The thesis contains a number of fundamental results on both well studied and ``new and exciting" graph parameters. The constituent works have already had significant impact in parameterized algorithms, dynamic algorithms, and graph theory. This is an absolutely outstanding thesis that not only deserves a doctoral degree, but should also be strongly considered for additional distinction.

Sincerely,

Daniel Lokshtanov
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Minor remarks:

Page 4: *The author of the thesis is not aware of any lower bounds against parameterized exact algorithms or parameterized constant-factor approximations for treewidth in time* $2^{o(k)} \cdot n^{O(1)}$,

Since the publication of this thesis a lower bound against 1.00000001 approximation in $2^{o(k)}$ time was given by Bonnet, please include citation.

Page 11 (top): We finally remark that the data structure for dynamic *rankwidth* …

Did you mean treewidth here?

Page 15: *without k-almost mixed minors in their adjacency matrix have their chromatic number bounded by* $2^{O(\log^{k-1} n)}$

The n in the exponent should probably be omega?