

Review of the PhD thesis of Marek Sokołowski entitled "Efficient Data Structures and Graph Width Parameters"

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September 24, 2024

Over the last two decades the research in the field of *parameterized complexity* managed to give a comprehensive overview of how certain (difficult in general) combinatorial optimization problems become tractable for specially structured instances. Specifically for graph problems the structural properties of the graph are often expressed as graph parameters and various algorithm for optimization problems over graphs of small value of a parameter are known.

The thesis reports on the success in extending the FPT paradigm to dynamic graphs. In the dynamic setting the edge set of the graph changes over time. The obtained results allow to algorithmically benefit from that the value of a parameter of the dynamic graph remains bounded.

Graph width parameters of interest to the author of the thesis come with corresponding graph decompositions. Given a graph one may compute a decomposition. Computing optimal decompositions is hard in general, but can be done assuming the existence of a small width decomposition. A typical algorithmic approach in the static setting is to compute an (approximately) optimal decomposition and then use it for (dynamic programming based) algorithms that utilize the decomposition.

An important aspect of the obtained results is that maintaining the proposed data structures for dynamically changing graphs is substantially less expensive than recomputing graph decomposition after the graph changes.

Treewidth. The first set of results is obtained for the standard parameter: the treewidth of the graph. The thesis presents a data structure that, given a target value of the parameter k , maintains a tree-decomposition of the graph. Whenever the true treewidth of the current graph is at most k , the maintained tree decomposition is of width at most $6k + 5$. In case the current width of the graph is too large, the data structure informs of this fact. Notably, the data structure returns to displaying a decomposition after the treewidth of the graph is small again.

Rankwidth. Rankwidth can be seen as an extension of treewidth to dense graphs. There exist simple dense graphs of large treewidth still allowing for a decomposition of bounded rankwidth. The author shows the existence of a data structure that can maintain a rank decomposition of the dynamic graph. This data structure works under the assumption that the rankwidth of the graph never exceeds the value of k and the maintained decomposition is of width at most $4k$.

Twinwidth. Twinwidth of a graph is a further generalization of the above two concepts. In particular planar graphs are of bounded twinwidth. More generally, twinwidth captures the limited complexity of graphs with excluded minors. This concept is more involved and related the existence of a d -twin-ordered adjacency matrix. The results in the thesis include a data structure to maintain properly ordered matrices, but also relations between the twinwidth and the chromatic number of the graph.

Evaluation. In my opinion the presented results are of great quality. They contribute to the area of highly studied graph parameters in a novel way. The extension of these decompositions to the dynamic setting appears very natural but challenging. It is clear that the author had become an expert in graph parameter's research.

Publications. The results presented in the thesis were published at top CS conferences including FOCS, STOC, and SODA. Overall, the candidate is a coauthor of 13 papers published at conferences and 2 journal publications.

Quality of presentation. Besides some very minor editorial issues, the thesis appears to be quite impressive as a document. Not only it contains over 200 pages of dense and high level scientific content, it also appears to be very well structured. Notably, it includes a part on research objectives, which contributes to the perception of the document as a consistent report from a systematic and well planned research project.

Conclusion. I am very positive about the thesis. It reports strong scientific results obtained through the execution of an ambitious research program. It definitely meets of the criteria of a good dissertation.

In my opinion it is an exceptionally good PhD thesis. I would like to propose that the PhD committee considers awarding it a distinction.

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