



To the Chairman of the
Academic Council of Computer and Information Sciences
Professor Andrzej Tarlecki
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Evaluation of the Dissertation
“Interesting Graph Problems”
by Mr. Marcin Smulewicz

The dissertation of Marcin Smulewicz is concerned with various algorithmic graph problems, ranging from optimization over decomposition to dynamic questions. These questions are tackled with techniques revolving around network flows, fast probabilistic evaluation of polynomials, fast matrix multiplication, as well as many diverse domain-specific ideas.

The **Introduction** of the thesis gives a brief overview over the different problems under investigation as well as the obtained results. **Chapter 2** lists the central graph-theoretic definitions needed in the main part of the dissertation. The main body of the dissertation begins after Chapter 2.

The first two Chapters are terse but informative and fulfill their purpose.

Chapter 3 considers many-visits TSP, where a complete graph $G = (V, E)$ with demands $k: V \rightarrow \mathbb{N}$ and edge lengths is given, and we need to compute a tour of minimum total length that visits each vertex $v \in V$ exactly $k(v)$ times. To keep the output compact, it is encoded as edge multiplicities, i.e., the number of times that each edge is traversed. The previously best known running times to solve this problem were [4]: $\mathcal{O}^*(5^n)$ and $\mathcal{O}^*(16^n)$ in polynomial space. Mr. Smulewicz improves these bounds as follows.

He provides a randomized algorithm in time $\mathcal{O}^*(2^n D)$ and polynomial space, where D denotes the maximum edge-length of the graph. This result is obtained by a combination of several techniques: The demands are trimmed to polynomial size, the permanent of the Edmonds matrix is evaluated in polynomial time over a field of characteristic 2 (where it equals the determinant), disconnected solutions are eliminated via a “Cut-and-Count” technique [10], the smallest degree monomials (corresponding to solutions) are recovered in randomized fashion, and Lagrange interpolation is used to construct a polynomial in terms of an edge-length variable, which adds a polynomial factor in D to the running time. By rounding, the algorithm can be turned into a $(1 + \varepsilon)$ -approximation with running time no longer depending on D . The achieved exponential dependence on n is best known already for the Hamiltonian cycle problem, i.e., without edge lengths and multiplicities.

Mr. Smulewicz further achieves an improved deterministic running time of $\mathcal{O}^*(4^n)$ via a tight analysis of the algorithm in [4]. Roughly, this algorithm iterates, via dynamic programming, over outbranchings that ensure connectivity of the solution, and solves the residual instances of multi-visit TSP (minus the outbranching) via min-cost flow. The key insight of the improved analysis lies in the fact that some subproblems can be pruned in the recurrence that defines the dynamic program.

Finally, Mr. Smulewicz presents an improved algorithm with running time $\mathcal{O}^*(7.88^n)$ in polynomial space. The algorithm of [4] computes in time $\mathcal{O}^*(4^n)$ the best outbranching for $\mathcal{O}^*(4^n)$ degree sequences for a total running time of $\mathcal{O}^*(16^n)$. The new idea is to integrate the deepest 4 layers of the outbranching into the min-cost flow computation, which reduces the size and the number of outbranchings to consider.

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Date
December 18, 2025

The results of Chapter 3 were published at ESA 2020 with 4 coauthors. In my estimation, the results are strong and significantly improve on the state-of-the-art. The randomized algorithm especially requires a series of insights and employs various algebraic methods.

In **Chapter 4**, the problem of finding the k -edge-connected components of a graph is investigated for $k = 4$. The main result is a linear time algorithm to solve this problem. Roughly, the algorithm decomposes the graph into 3-edge-connected components, finds all 3-edge-cuts, and reconstructs a partition into 4-edge-connected components from these cuts. The key ingredient to this algorithm is the method of enumerating 3-edge-cuts. For this, an arbitrary DFS-tree is fixed and it is observed that for every tree edge on a cut with at most two tree edges, there are only constantly many candidates for the other edges on the cut. The challenging cuts are those composed of 3 tree edges, which are determined by contracting non-tree edges and recursing.

The result of Chapter 4 was published at ESA 2021 with 3 coauthors and received the best student paper award. In my estimation, the linear time algorithm for $k = 4$ is a strong result on a prominent problem that received significant attention at the time with the same result concurrently published at the same conference [21]. By now, a linear time algorithm is known for all constant k [31], which was a celebrated result.

Chapter 5 deals with fully dynamic algorithms to maintain strong connectivity. A data structure with update time $\mathcal{O}(n^{1.528})$ to maintain strongly connected components is presented, a bound that was only known in an amortized sense before. The result employs the reachability data structure of [41] for auxiliary graphs to maintain, for subsets of the vertices, the information (a bit), whether pairs lie in the same strongly connected subgraph. Sufficiently many such auxiliary graphs are built to be able to identify strongly connected components by bit signatures.

Additionally, a data structure with update time $\mathcal{O}(n^{1.406})$ to maintain whether the graph is strongly connected is presented. Per update, this data structure uses $\tilde{\mathcal{O}}(1)$ reachability queries of the reachability data structure of [46]. It maintains representatives for components that are a source or sink in the graph of strongly connected components. These representatives suffice: When an edge is added, representatives may have to be eliminated and when an edge is deleted, it suffices to consider its endpoints as new representatives.

The results of Chapter 5 were published at ESA 2023 with 1 coauthor and were later improved for planar graphs by the same authors in ICALP 2024 (not formally included in the thesis). While the results do not seem as technically involved as the ones in the previous chapters, both results are conditionally tight. I therefore again consider the results to be strong.

Finally, **Chapter 6** is concerned with partitioning a graph into smaller graphs whose maximum average degrees add up to the original one. Mr. Smulewicz proposes a polynomial algorithm that partitions the graph into a graph of bounded degeneracy and a graph of reduced maximum average degree. The main result is the existence of such a partition, with the algorithm serving as a constructive proof. The method is based on extracting certain vertices from the flow network used for computing the maximum average degree.

The result of Chapter 6 was published in the Electronic Journal of Combinatorics with 1 coauthor. While the result only partially solves the question of whether the maximum average degree is partitionable, it unifies several well-known partitions of planar graphs and strengthens a known result [15] for partitioning a graph into an independent set and a forest of bounded degree, by restricting to forests with small connected components. The result has lower technical difficulty and impact than the results of previous chapters, but still constitutes significant progress.

Overall, I evaluate the results of the dissertation as very strong and very clearly sufficient to grant a PhD. The results have led to 4 publications overall, including 3 strong publications at the European Symposium on Algorithms (ESA), one of which received a best student paper award. It should also be mentioned that the publications are with different coauthors and 2 publications are with only a single coauthor, which emphasizes the contribution of the candidate to these results.

In terms of methods, I am particularly impressed by the wide spectrum of techniques, ranging from flow based methods, over algebraic methods to structural analyses of the specific domains.

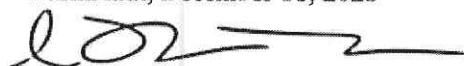
Since the thesis is cumulative, there is not much to be said about writing and presentation.

On the negative side, the thesis covers relatively few publications over a relatively long time period, and it does not include publications at international top venues such as SODA, STOC, FOCS. Therefore, I cannot recommend a distinction (*summa cum laude*).

Based on this evaluation, I award the dissertation the grade

very good (*magna cum laude*).

Darmstadt, December 18, 2025



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