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Report on Jędrzej Kołodziejski's doctoral thesis.

This thesis studies several aspects of modal logic (ML), and makes several contributions to the fields of model theory, verification, and automata theory. The document is very nicely written, with many examples illustrating the definitions and results. Moreover, the broadness of the technical tools employed shows that the author is able not only to navigate a high level of technicality, but also to connect his research with several adjacent fields, and to make use of a various array of both classical and recent results. This witnesses very promising curiosity and advanced research maturity.

The first part of the thesis introduces the many notions that will be used throughout the thesis, together with basic results linking these notions. This includes two-player games, automata, and ML with fixed points extensions. The contributions are then mainly split into two chapters (chapters 3 and 4).

Chapter 3 proposes a model-theoretic study of ML, with a focus on bisimulational categoricity. While categoricity (i.e. the property of having a unique model) is well-studied up to isomorphism, the natural question of categoricity up to bisimulation was not thoroughly investigated in the literature. The thesis proposes many results in this direction, some of them with a high level of technicality and creative arguments. Equivalence between bisimulational categoricity and the existence of an image-finite (or finite) model are given in four cases: over all models, over two-way models, over transitive models, and over ordinal ones. Moreover, the author takes care of identifying how general are the arguments provided, by finding counter-examples whenever possible, and giving specialized arguments when necessary. For instance compactness of the logic over the class of models considered seems to be a key property, but it is shown that it is not sufficient to guarantee that the theorem generalizes to any setting where the logic is compact. In the study of compactness properties, I especially enjoyed the proof that ML is compact over ordinal models: appealing to Higman Lemma is an elegant use of an unexpected tool in this setting.

Chapter 4 goes a different route and introduces countdown mu-calculus, an extension of mu-calculus where the number of fixed points unfolding is

explicitly bounded by ordinals. As mentioned in the chapter introduction, this can be placed in the line of work extending MSO with boundedness operators such as MSO+U or cost MSO. Here, the classical equivalences mu-calculus/games/automata are extended to the countdown setting, by introducing games and automata with ordinal counters, and showing that they are expressively equivalent to countdown mu-calculus. Moreover, it is shown that contrarily to the classical case, the vectorial version of countdown mu-calculus is more expressive than the scalar one. This is a highly technical proof, and one of the impressive contributions of this work. Automata models are also provided to match vectorial mu-calculus, and the fine-grained complexity of these models are compared, by matching parameters of (several variants of) automata to fixed points nestings in formulas. Moreover, it is shown that this common parameter yields a strict hierarchy, and is therefore a relevant measure of language complexity. Finally, decidability of countdown mu-calculus satisfiability is conjectured, and is obtained in special cases, the most involved being the Büchi case allowing for one nesting of fixed points. I think it is particularly valuable that in addition to make several interesting and highly involved contributions, the thesis offers via this conjecture a new line of research that can prove to be fruitful in the future.

The document concludes by summing up the results and mentioning avenues that were not taken in the document but could easily have been. In particular, many results could have been formulated in a more general coalgebraic setting, allowing to lift them to other structures. I personally agree with the choice made by the author to favour clarity over generality, especially when the coalgebraic generalisation is orthogonal to the proof techniques used in the thesis and can be obtained "for free" as mentioned.

Overall, I believe the thesis is excellent and largely sufficient to grant a PhD to Jędrzej Kołodziejski. I also propose a deliberation of the committee to award this work with an honorary distinction.

I join in a separate file a list of detailed remarks and typos, those are purely indicative and may be of help to prepare a final version.

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