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Review of the Dissertation of Jana Masaříková

To whom it may concern,

it is my pleasure to review the dissertation "Computational Complexity of Combinatorial Problems in Hereditary Graph Classes" by Ms. Masaříková.

Synopsis

The dissertation, which is based on five publications, studies the structure of hereditary graph classes and its algorithmic application for several well-studied problems in computer science.

Besides an introduction into the area, a summary of the results, and their relation to previous work (Chapter 1) and a recall of the necessary basic concepts in graph theory and computational complexity theory, the work consists of five main chapters.

In Chapter 3, the MAXIMUM WEIGHT INDEPENDENT SET problem is studied in graphs that do not contain long claws (more precisely graphs that are $S_{t,t,t}$-free which is the graph obtained by subdividing each edge of a claw $t-1$ times). The first main algorithmic result is an algorithm that outputs either such a subdivided claw or a vertex set P of logarithmic size such that deleting P leaves a graph with a very well-structured extended strip decomposition. The second main result is an application of the first result to show an improved subexponential-time algorithm for MAXIMUM WEIGHT INDEPENDENT SET on $S_{t,t,t}$-free graphs. This is also generalized to graphs that have no induced subgraph that consists of several but few independent $S_{t,t,t}$-free graphs.

Chapter 4 studies the 3-COLORING problem on graphs that have no induced $2P_4$ and no induced C_5. This is at least in part motivated by the fact that the complexity of 3-COLORING on $2P_4$-free graphs. The work makes progress towards answering this open question by showing a polynomial-time algorithm for the $(2P_4, C_5)$-free graphs. The algorithm is based on a set of reduction rules that either remove parts of the graph, extend a partial coloring, or exclude some possible colors for some vertices. The main difficulty is to show that in the case where the graph contains a C_7 that after an initial coloring of that C_7, the reduction rules produce either some components that are trivial or that can be solved via reduction to 2-SAT.

Chapter 5 now studies for certain graph classes defined by two forbidden induced subgraphs how the cliquewidth of these graph classes behaves. More precisely, it is studied whether the graphs from that class and whether the atoms from that class (these are graphs without a clique separator) have unbounded cliquewidth. This knowledge is useful as for several important problems, bounded cliquewidth for atoms is sufficient to give polynomial-time algorithms. A main contribution is that the class of $(2P_2, \overline{P_2 + P_3})$-free graphs has this property. Further, many new examples of graph classes with unbounded cliquewidth even on the atoms are given. The results bring us closer to a dichotomy concerning the (atom)-cliquewidth for (H_1, H_2)-free graphs. Near the end, the chapter also lists all open cases.

In Chapter 6, an FPT-algorithm for the Bipartite Permutation Vertex Deletion problem parameterized by the number k of vertex deletions is shown. This algorithm is based on the forbidden subgraph characterization, but the challenge is that the forbidden subgraphs can be arbitrarily large. The approach taken in the work is to exploit the structure of almost bipartite permutation graphs which are graphs which do not contain the forbidden subgraphs of size up to 9. Informally, it is shown that the holes of such graphs are dominating sets, that the other vertices have only very restricted neighborhoods in shortest holes and large parts of the graph already fulfill the properties of bipartite permutation graphs. This gives a a polynomial-time algorithm for Bipartite Permutation Vertex Deletion problem on almost bipartite permutation graphs via a polynomial number of calls of a Min Cut algorithm. That algorithm then implies the above-mentioned FPT algorithm via standard techniques.

The final chapter, Chapter 7, studies the relation of edge-disjoint triangle packings and edge-sets hitting all triangles in the class of threshold graphs and even-balanced co-chain graphs. More precisely, Tuza's conjecture which states that the triangle hitting size is at most two times the maximum size of a triangle packing is shown to hold for these graph classes. For the threshold graphs, which are essentially split graphs with a chain graph between independent set and clique part, the proof relies on constructing a large packing of triangles using the fact that, roughly speaking, there are many vertices in the clique part which have many neighbors in the independent set part. A somewhat similar approach is used for even-balanced co-chain graphs where we have two cliques again with a chain graph between them.

Assessment

In my opinion, the dissertation submitted by Ms. Masaříková makes significant progress in several interesting and important areas of graph algorithms.

The results in Chapter 3 substantially extend our knowledge on the classes on which Maximum Weight Independent Set can be solved more efficiently. The tool used for these results can become quite useful in further work on this graph classes with one hope maybe that other tractability results for claw-free graphs can be extended to $S_{t,t,t}$. The idea to make use of parts of the extended strip decompositions in the structural result is quite innovative and the application of the structural result for computing independent sets is also non-trivial even if it builds on previous work.

In Chapters 4 and 5, the results rely on a deep understanding of the structure of the neighborhoods of short induced cycles in the considered graph classes. For the coloring problem, it is shown how this structure interacts with the presented reduction rules; for the cliquewidth bounds it is shown that this can be used to give compact cliquewidth expressions. Both results again make substantial progress in the two considered areas, bringing us closer to resolutions of the open problem for 3-Coloring on $2P_4$-free graphs and to a full characterization of cliquewidth bounds for bigenic graph classes and their atoms.

In Chapter 6, the FPT algorithm for this challenging problem is highly nontrivial. It required a deep understanding of the structure of almost bipartite permutation graphs which may be helpful in future research on this type of graphs, for example when it comes to studying the complexity of other problems on this graph class. Finally, Chapter 7 also makes some maybe small but still nontrivial progress on a classic and very hard graph-theoretic conjecture whose resolution could find several algorithmic applications, for example for improving kernelization or approximation bounds.

The thesis is also well-written: All the technical details are presented in a mathematical rigorous way with very helpful informal expositions before the technical details. Useful figures are given throughout the thesis, in particular for the nontrivial constructions of graph classes with unbounded cliquewidth and the structure of almost bipartite permutation graphs. The introduction gives a comprehensive survey of the previous work, the motivation for the new results, and the approaches that were used to obtain them.

It should also be mentioned, that the results contained in Ms. Masaříková's dissertation have been presented at high-ranking conferences with very competitive peer review (for example ICALP and WG) and received a best paper award at IPEC 2021.

Summarizing, I deem this thesis as **sufficient to grant a PhD**. I would also like to propose that **this excellent work should be awarded an honorary distinction.**

With kind regards,

Prof. Dr. Christian Komusiewicz