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University of Warsaw
Faculty of Mathematics, Informatics and Mechanics

Exam for 2nd cycle studies of MACHINE LEARNING

July 1, 2025

Solving time: 150 minutes

In each of the 30 problems there are three variants: (a), (b), and (c). For each variant you should answer if it is true, writing **YES** *or* **NO** *in the box close to it. To correct your answer you should cross out the box and write the correct word on its left side.*

Example of a correctly solved problem

4. Every integer of the form $10^n - 1$, where n is integer and positive,

$\boxed{\texttt{YES}}$ (a) is divisible by 9;

$\boxed{\texttt{NO}}$ (b) is prime;

$\boxed{\texttt{YES}}$ (c) is odd.

You can write only in the indicated places and only the words **YES** *and* **NO**. *Use pen.*

Scoring

You get "big" points (0 – 30) and "small" points (0 – 90):

- *one "big" point for each problem, in which you correctly solved all three variants;*
- *one "small" point for each correctly solved variant. So 3 "small" points in a single problem give one "big" point.*

The final result of the exam is the number

$$W = \min(30, D + m/100),$$

where D is the number of "big" points, and m is the number of "small" points, e.g. score 5.50 means that a candidate correctly solved 50 variants in the whole test, but gave correct answers to all three variants in a set of some five problems.

"Big" points are more important. "Small" points are just to increase resolution in the case when many candidates get the same number of "big" points.

Good luck!

1. Every convergent sequence of reals

☐ (a) is monotonic;
☐ (b) has a monotonic subsequence;
☐ (c) has a lower bound.

2. Series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[n]{n}}$

☐ (a) is absolutely convergent;
☐ (b) has increasing sequence of partial sums;
☐ (c) has convergent sequence of partial sums.

3. Let A be a 2025×2025 matrix over $\mathbb{R}$ of rank 2025. It follows that

☐ (a) the image of A is $\mathbb{R}^{2025}$;
☐ (b) the image of A^{T} is $\mathbb{R}^{2025}$;
☐ (c) A^{T} is the matrix of a monomorphism $\mathbb{R}^{2025} \to \mathbb{R}^{2025}$.

4. Let A be a 2025×2025 diagonal matrix over $\mathbb{C}$ such that the elements on the diagonal are all the 2025th roots of unity. It follows that

☐ (a) A is nonsingular;
☐ (b) 1 is an eigenvalue of A;
☐ (c) -1 is an eigenvalue of A.

5. Let $L(U, V)$ denote the space of linear maps from U to V. Let X and Y be linear spaces over $\mathbb{R}$, with $\dim X = 4$ and $\dim Y = 7$. It follows that

☐ (a) $\dim L(X, Y) = 28$;
☐ (b) $\dim L(Y, X) = 11$;
☐ (c) $\dim L(L(Y, Y), L(X, \mathbb{R})) = 196$.

6. For all complex numbers a and b,

☐ (a) if $|a| = |b|$, then $a = b$ or $a = -b$;
☐ (b) if $|a| + |b| = 0$, then $a = b = 0$;
☐ (c) if $a^2 + b^2 = 0$, then $a = b = 0$.

7. The cardinality of the inverse image of a set $A \subseteq \mathbb{N}$ under a function $f \colon \mathbb{N} \to \mathbb{N}$ is

☐ (a) equal to the cardinality of A;
☐ (b) less than or equal to the cardinality of A;
☐ (c) greater than or equal to the cardinality of A.

8. Composition of two partial orders on $\mathbb{N}$ is

☐ (a) a partial order;
☐ (b) an uncountable set;
☐ (c) a nonempty set.

9. For $n \geq 0$ the sum $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} 1$ is

- [ ] (a) less than or equal to $n(n-1)(n-2)$;
- [ ] (b) equal to $\sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=j}^{n} 1$;
- [ ] (c) greater than or equal to $\binom{n+2}{3}$.

10. In an undirected graph G with the set of vertices $\{1, 2, \ldots, 100\}$ an edge $\{m, n\}$ exists if and only if $m \neq n$ and $m \cdot n$ is an even number. The number of all triangles (subgraphs isomorphic to K_3) in G equals

- [ ] (a) $\binom{100}{3} - \binom{50}{3}$;
- [ ] (b) $\binom{50}{3} + 50 \cdot \binom{50}{2}$;
- [ ] (c) $\frac{1}{2} \cdot \binom{100}{3}$.

11. The number of ways to distribute 10 indistinguishable cookies to Alice, Bob, Charlie and David so that each person receives an odd number of cookies is equal to

- [ ] (a) the coefficient of x^{10} in the power series expansion of the expression $\frac{x^4}{(1-x)^4(1+x)^4}$;
- [ ] (b) the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 = 3$ in nonnegative integers;
- [ ] (c) the number of partitions of a set of size 10 into 4 blocks of odd size.

12. Consider integer n such that $n \geq 100$, $\binom{n}{2}$ is even and $\binom{n}{3}$ is divisible by 3. It follows that

- [ ] (a) n is divisible by 2 or n is divisible by 3;
- [ ] (b) $\binom{n}{5}$ is divisible by 5;
- [ ] (c) $\binom{n}{6}$ is divisible by 6.

13. Let $\mathrm{P}(B_1) = \frac{1}{4}$, $\mathrm{P}(B_2) = \frac{3}{4}$, $\mathrm{P}(A|B_1) = \frac{1}{2}$, $\mathrm{P}(A|B_2) = \frac{1}{3}$. It follows that

- [ ] (a) $\mathrm{P}(A \cap B_1) = \frac{1}{8}$;
- [ ] (b) $\mathrm{P}(B_1 \cup B_2) = 1$;
- [ ] (c) $\mathrm{P}(B_2|A) \geq \frac{1}{4}$.

14. Let X and Y be random variables with values from the set $\{0, 1, 2\}$. The following condition is sufficient for the independence of X and Y

- [ ] (a) $\mathrm{P}(X = 1) \cdot \mathrm{P}(Y = 2) = \mathrm{P}(X \cdot Y > 0)$;
- [ ] (b) $\mathrm{E}(X + Y) = \mathrm{E}(X) + \mathrm{E}(Y)$;
- [ ] (c) $\mathrm{P}(X = x \wedge Y = y) = \mathrm{P}(X = x) \cdot \mathrm{P}(Y = y)$ for all $x, y \in \{0, 1, 2\}$.

15. In a (time-homogeneous) Markov chain with three states 1, 2, 3, all of which are recurrent, the mean recurrence time for state 1 is 2, and the mean recurrence time for state 2 is 4. It follows that

- [ ] (a) the mean recurrence time for state 3 is less than or equal to 6;
- [ ] (b) the mean recurrence time for state 3 is greater than or equal to 6;
- [ ] (c) this Markov chain is aperiodic.

16. Let X be a continuous random variable with probability density function $f_X\colon \mathbb{R} \to \mathbb{R}$, strictly increasing cumulative distribution function $F_X\colon \mathbb{R} \to [0;1]$ and finite expected value. For the random variable $Y = -X$, let f_Y be the probability distribution function of Y, and F_Y be its cumulative distribution function. It follows that

- [ ] (a) $f_Y(t) = f_X(-t)$ almost everywhere;
- [ ] (b) $F_Y(t) = 1 - F_X(-t)$;
- [ ] (c) $\mathrm{E}(Y) = -\mathrm{E}(-X)$.

17. A researcher constructs an ML test that classifies patients. The test result is either positive, when the patient is classified as sick, or negative, when the patient is classified as healthy. The sensitivity of the test is 0.8 and its specificity is 0.7. It follows that

- [ ] (a) the test identifies 80% of healthy patients as healthy;
- [ ] (b) 70% of the patients classified as healthy are indeed healthy;
- [ ] (c) the test correctly classifies 80% of the patients.

18. The standard k-means algorithm with a parameter k for n observations returns

- [ ] (a) a partition of observations into k groups with the lowest variances of distances within groups;
- [ ] (b) a set of groups such that their middle points are observations from the input data;
- [ ] (c) the same results in different runs on the same input data.

19. Kolmogorov–Smirnov test

- [ ] (a) tests whether an input sample comes from a normal distribution of given parameters;
- [ ] (b) tests whether an input sample comes from a normal distribution of unknown parameters;
- [ ] (c) assumes that an input sample comes from a distribution of finite variance.

20. When testing multiple statistical hypotheses, the false discovery rate (FDR) is

- [ ] (a) the probability of making type I error in at least one of the tests;
- [ ] (b) the estimated proportion of falsely rejected null hypotheses at the given significance level;
- [ ] (c) the product of all p-values and the number of tested hypotheses.

21. For infinitely many n there exists a n-node red-black tree with

- [ ] (a) the number of red nodes greater by 1000 than the number of non-null black nodes;
- [ ] (b) the number of non-null black nodes 3 times greater than the number of red nodes;
- [ ] (c) the number of nodes on the longest path from the root to a leaf 1.5 times greater than the number of nodes on the shortest path from the root to a leaf.

22. Suppose that a disjoint-set (union-find) forest, with union by size and path compression, contains n elements. It follows that

☐ (a) every tree in the forest is a binary tree;

☐ (b) the worst-case time of `find` operation is $\Theta(\log n)$;

☐ (c) the total time of any sequence of m `find`/`union` operations is $O(n + m)$.

23. Table `T` was defined as

```
CREATE TABLE T (A INT NOT NULL, B INT NOT NULL)
```

and its schema was never modified. The following query is issued:

```
SELECT A, SUM(B) FROM T GROUP BY A
```

It follows that

☐ (a) the number of rows returned by the query is greater than or equal to the number of rows in table `T`;

☐ (b) the number of rows returned by the query is less than or equal to the number of rows in table `T`;

☐ (c) the sum of all values in the second column of the result of the query is equal to the sum of all values in column `B` in table `T`.

24. The solution to the mutual exclusion problem for two processes, `P1` and `P2` presented on the right, running on in-order processors which observe a consistent view of shared memory

☐ (a) ensures that `P1` does not enter the critical section if `P2` is already in the critical section;

☐ (b) may lead to a deadlock;

☐ (c) ensures that `P1` and `P2` enter the critical section in alternation.

```
int x = 0;
int y = 0;
int z = 0;

process P1 {
    for (;;) {
        local_section();
        x = 1;
        z = 0;
        while (y != z) {}
        critical_section();
        x = 0;
    }
}

process P2 {
    for (;;) {
        local_section();
        y = 1;
        z = 1;
        while (x == z) {}
        critical_section();
        y = 0;
    }
}
```

25. It is possible to receive a message after the process that sent it has terminated

- [ ] (a) in the direct asynchronous communication model;
- [ ] (b) in the direct synchronous communication model;
- [ ] (c) in indirect communication via a tuple space.

26. Consider the Python 3 code on the right. Executing this code

- [ ] (a) will print [1, 2, 3, 4];
- [ ] (b) after uncommenting line 3 will print [1, 2, 3, 4];
- [ ] (c) after uncommenting line 4 will print [1, 2, 3, 4].

```
1 a, b = [1, 2, 3], []
2 a.append(4)
3 #b = a
4 #b = a + [5, 6]
5 b.append(5)
6 print(a)
```

27. Consider the Python 3 code on the right. It will run correctly and print 2 after uncommenting the definition and call of function

- [ ] (a) `f1`;
- [ ] (b) `f2`;
- [ ] (c) `f3`.

```
x = 0

#def f1():
#    x += 2

#def f2():
#    global x
#    x = 2

#def f3():
#    if False:
#        x += 2

#f1()
#f2()
#f3()

print(x)
```

28. Consider the Python 3 code on the right. It will run correctly and print 1 after uncommenting line

- [ ] (a) 8;
- [ ] (b) 9;
- [ ] (c) 10.

```
1  class A:
2      def __init__(self):
3          self.x = 1
4          self._x = 1
5          self.__x = 1
6
7  a = A()
8  #print(a.x)
9  #print(a._x)
10 #print(a.__x)
```

29. The network address

- [ ] (a) 10.10.0.1 is a globally routable IP address;
- [ ] (b) 127.0.0.1 is a loopback address;
- [ ] (c) 255.255.255.255 is a broadcast address.

30. In computer memory, starting from address A, six consecutive 8-bit bytes hold the following values: 6, 5, 4, 3, 2, 1. The value of the 16-bit number stored in standard binary format at address

- [ ] (a) A, in a big-endian architecture, is 65;
- [ ] (b) $A + 2$, in a little-endian architecture, is 1028;
- [ ] (c) $A + 4$, in the x86 architecture, is 258.