

40th Autumn School in Algebraic Geometry

Binomial Ideals

ŁUKĘCIN, POLAND, 3 – 9 SEPTEMBER 2017

Binomial ideals arising from combinatorics

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No special knowledge will be required to understand the talks. However, it is highly desirable for the participants to be familiar with fundamental materials on ideals of the polynomial ring as well as on partially ordered sets. More precisely, the participants are strongly requested to read the following subsections of [HH] in advance:

- 1.1.1 The K -basis of a monomial ideal
- 1.5.1 Simplicial complexes
- 9.1.1 Basic definitions (up to Lemma 9.1.1 and its proof)
- 9.1.2 Finite partially ordered sets (up to Theorem 9.1.7 and its proof)

Monday 04 September 2017

A quick introduction to Gröbner bases

- Dickson's Lemma
- Monomial orders, initial ideals and Gröbner bases
- Hilbert's basis theorem
- The division algorithm
- Buchberger's criterion and Buchberger's algorithm

Tuesday 05 September 2017

Toric rings and toric ideals

- Integer matrices and toric ideals
- Toric rings and toric ideals
- Toric ideals of finite graphs

Wednesday 06 September 2017

Join-meet ideals of finite lattices

- Review on classical lattice theory
- Gröbner bases of join-meet ideals
- Join-meet ideals of modular non-distributive lattices

Thursday 07 September 2017

Normal polytopes and regular unimodular triangulations

- Foundations on convex polytopes
- Integer decomposition property and normal polytopes
- Regular triangulations and unimodular triangulations
- Normal edge polytopes

Friday 08 September 2017

Order polytopes of finite partially ordered sets

- Combinatorics on order polytopes
- Toric rings of order polytopes
- Toric ideals of order polytopes
- Triangulations of order polytopes

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