DS-optimal designs for RCR models with heteroscedastic errors

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Assume that we have a linear random coefficient regression model with heteroscedastic errors. That is, we consider the model

$$y(x) = \beta_1 + \beta_2 x + e(x),\ x \in [a, b], \tag{1}$$

where $[a, b] \subset \mathbb{R}$ is a design region, $\beta = [\beta_1\ \beta_2]^T$ is a random vector uncorrelated with $e(x)$, $\mathbb{E}[e(x)] = 0$, and $\mathrm{cov}[e(x)] \propto \frac{1}{\lambda(x)}$ with $\lambda : \mathbb{R} \to \mathbb{R}$ being a given positive function. Also assume that according to this model we have uncorrelated observations $y(x_i)$ at $x_i,\ i = 1, \ldots, n$, respectively.

In this talk we examine the problem of finding optimal designs for estimation of β and $\mathbb{E}[\beta]$ in (1). Each continuous k-point design ζ is a discrete probability measure taking values $\omega_i \geqslant 0$ at distinct points $x_i \in [a, b],\ i = 1, \ldots, k$. Considered criteria of optimality include several classical criteria (D-, A-, and E-optimality) as well as DS-optimality introduced by Sinha [2,3]. If the parameter of interest is denoted by θ, then a design ζ is said to be DS-optimal for the estimator $\hat{\theta}(\zeta)$ if it maximizes the probability $\mathbb{P}(\|\theta - \hat{\theta}(\zeta)\| < \epsilon)$ for all $\epsilon > 0$.

Our work covers two main results. Firstly, in similar but more general fashion than in [1], we provide and discuss conditions for the model (1) which are sufficient if we want to reduce any k-point design ($k \geqslant 2$) into a 2-point design but keep quality (in terms of the above optimality criteria) of the former. Secondly, we provide sufficient and necessary conditions for existence of the DS-optimal design among 2-point designs.

References

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