Single-use transducers for data words

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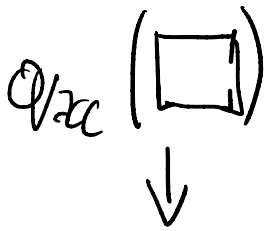
Register automata (by example)

The first letter appears again



3 2 2 1 3 2

$\mathbb{A} = \langle \mathbb{N}, = \rangle$

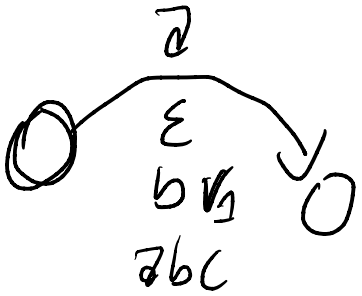
Register transducers

Mark all the positions with the first letter with a. ($A^* \to \{a, b\}^*$)

3 2 2 1 3 3

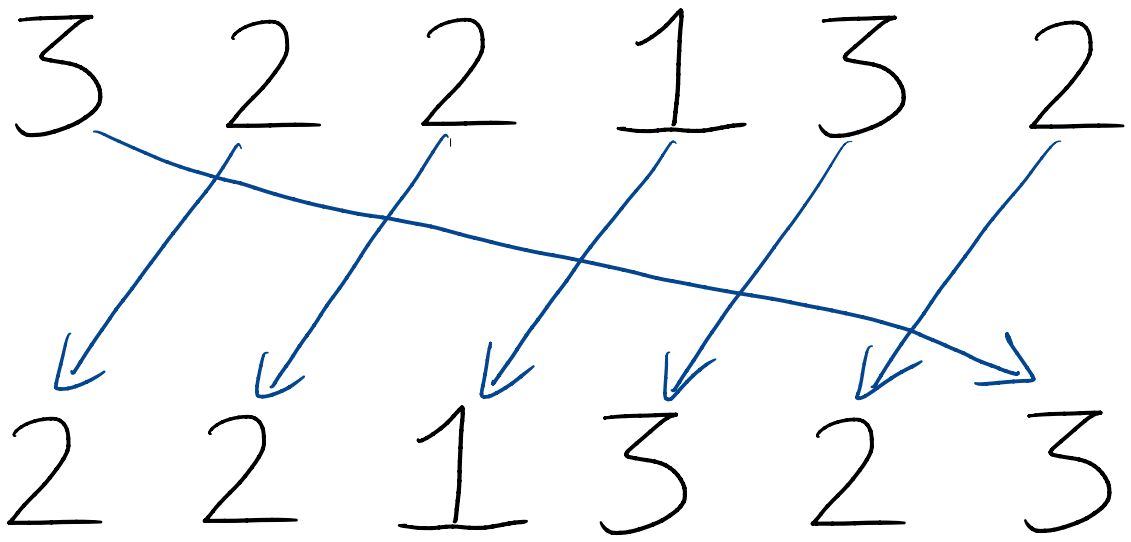
↓

a b b b a a



Register transducers

Cyclic shift by 1 ($/A^* \to /A^*$)



Register two-way transducers

Reverse

3 2 2 1 3 2

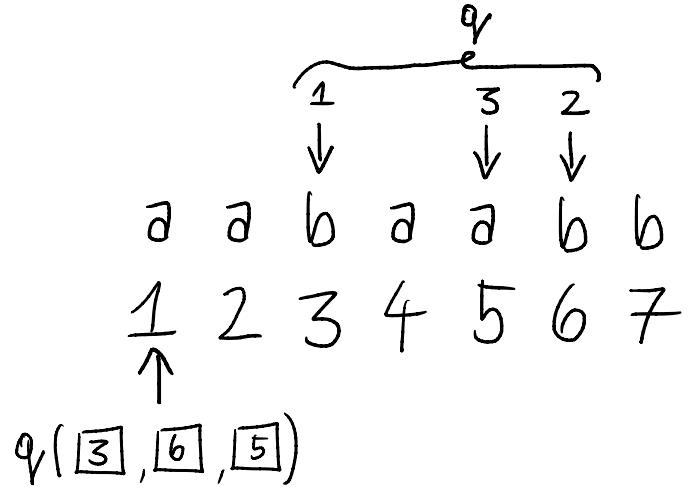
2 3 1 2 2 3

Register two-way automata have undecidable emptiness

On words over $\Sigma \times \mathbb{A}$, where every two atoms are distinct

(Σ: finite)

a 2-way register automaton can simulate a 2-way multi-headed automaton on Σ.



Emptiness of two way register automata	$\geq$	Emptiness of LOG-SPACE Turing Machines

(Emptiness of LOG-SPACE Turing Machines: undecidable)

Register SST transducers

Two types of registers:

1) Atom (control) registers
2) String (output) registers

Subject to the copyless restriction

$S_1 := S_1 r_1 a S_2$

$S_2 := a$

$S_3 := \varepsilon$

update example

$S_1 := S_2 S_1$

$S := S a$

reverse

Register SST transducers are not closed under compositions

1) "The last letter appears again" is not recognised by deterministic register automata.

Suppose that it is reconised with 7 registers.

Consider the word:

In this position...

1 2 3 4 5 6 7 8 → 3
                → 15

2) Functions $A^* \to \{Yes, No\}$ are equivalent to (one-way) register automata

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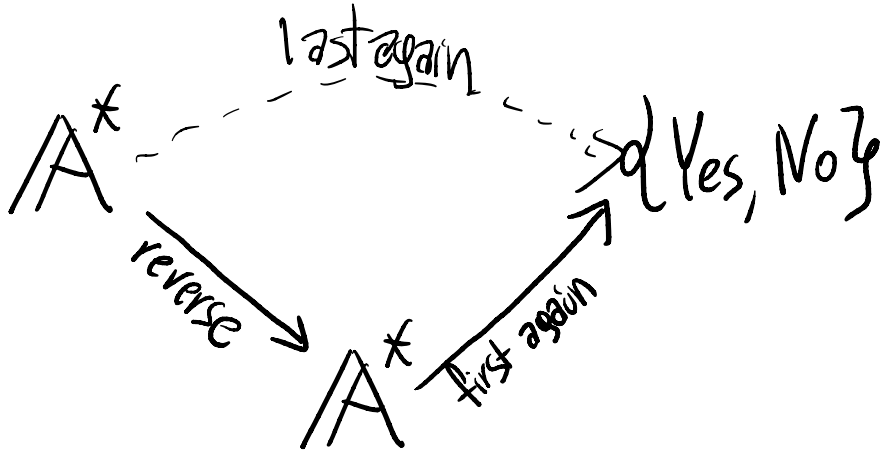
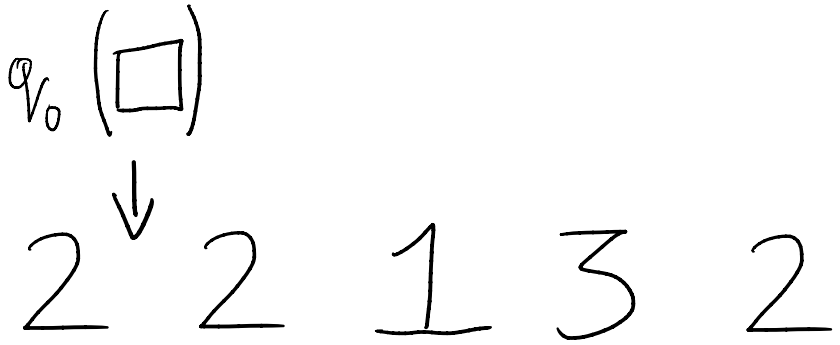


Single-use restriction

Whenever we compare value of a register with other value, we lose it.

a non-example

The first letter appears again



Single-use transducers

With the single-use restriction:

1) Register SSTs and two-way transducers are equivalent.

2) They are closed under composition.

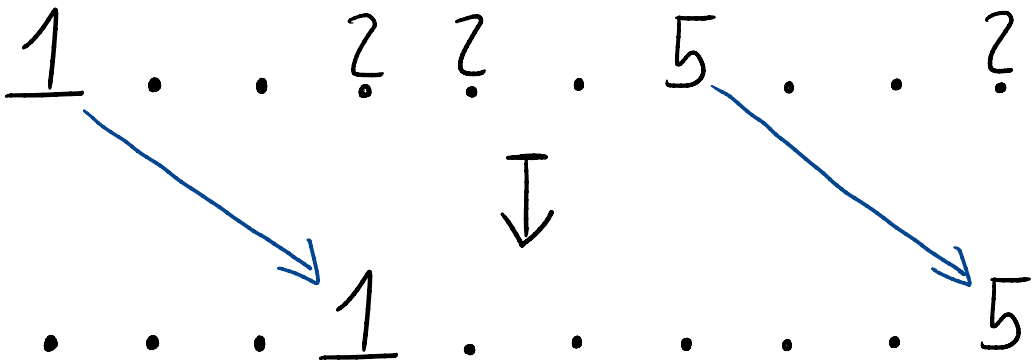
3) Have decidable equivalence.

4) Admit a Krohn-Rhodes decomposition

Krohn–Rhodes decomposition

Single-use 2-way transductions can be decomposed into:

- the prime functions for two-way transductions over finite alphabets (as defined in Mikołaj's talk)
- Single-use atom propagation:



Perspectives:

1) Other atoms (e.g. $\langle \mathbb{Q}, \leq \rangle$)
2) Abstract set of states, connections with linear types
3) Register automaton $\xrightarrow{?}$ Single-use register automaton
4) Unambiguous single-use automata
5) MSO transductions — rigidly guarded

Thank you!