

Single-use automata and orbit-finite monoids for total-order atoms

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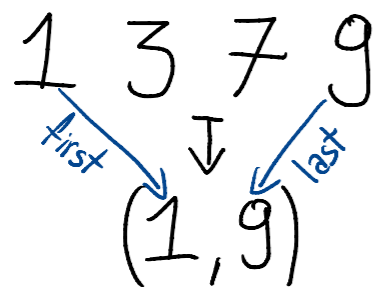
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Observation: There are such monoids M and N , that M has finite γ -height, N has infinite γ -height, and $N \subseteq M$

Letters are strictly increasing

This language is recognised by the following morphism



Its monoid is defined as follows:

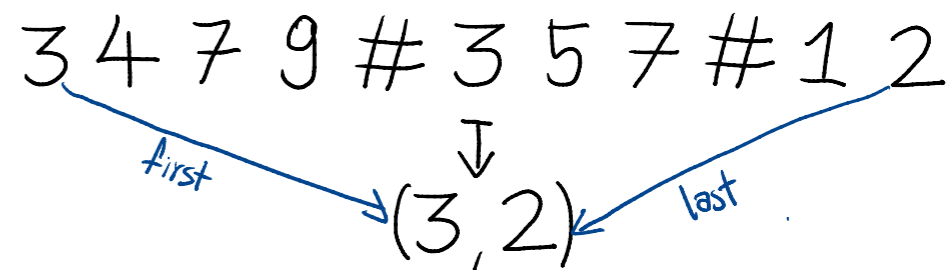
$$N = \mathbb{Q}^2 + \perp + 1$$

$$(a_1, b_1) \cdot (a_2, b_2) = \begin{cases} (a_1, b_2) & \text{if } a_2 < b_1 \\ \perp & \text{otherwise} \end{cases}$$

It has an infinite infix chain: $(-1, 1), (-2, 2), (-3, 3), \dots$

Letters between #'s are strictly increasing

The morphism is almost the same:



The monoids are also very similar:

$$M = (\mathbb{Q} + \#)^2 + \perp + 1$$

$$(a_1, b_1) \cdot (a_2, b_2) = \begin{cases} (a_1, b_2) & \text{if } a_2 < b_1 \text{ or } a_2 = \# \text{ or } b_1 = \# \\ \perp & \text{otherwise} \end{cases}$$

But now every two elements from $(\mathbb{Q} + \#)^2$ are each other's infixes:

$$(3, \#) \cdot (x, y) \cdot (\#, 2) = (3, 2)$$