(Joint w/ Koehler, Shamir, Zeitouni).

Motivation: The Gaussian setting.

γ - standard Gaussian on $\mathbb{R}^n$.

If $d\nu = e^V d\gamma$, $\nabla^2 V \preccurlyeq (1-\delta) Id$.

$\Rightarrow$ ν satisfies:

(i) $\forall \varphi$ 1-Lip, $Var_\nu[\varphi] \le \frac{1}{\delta}$

(ii) Poincaré: $Var_\nu[\varphi] \le \frac{1}{\delta} \mathcal{E}_\nu(\varphi,\varphi) := \mathbb{E}_\nu |\nabla\varphi|^2$.

(iii) Log-Sobolev:

$$\mathbb{E} f^2 = 1 \Rightarrow \mathbb{E} f^2 \log f^2 \le \frac{1}{\delta} \mathcal{E}(\varphi,\varphi).$$

In this talk: $\gamma \longrightarrow \mu :=$ Uniform on $\{-1,1\}^n$.

$$\partial_i \varphi(x) := \frac{\varphi(x_{(i\to +1)}) - \varphi(x_{(i\to -1)})}{2}.$$

$$\nabla\varphi = (\partial_1\varphi, \dots, \partial_n\varphi).$$

$$Lip(\varphi) = \max_{i,x} |\partial_i \varphi(x)|.$$

Here $\int |\nabla \varphi|^2 \, d\nu$ not a Dirichlet form.

We'll use the dirichlet form $\mathcal{E}$ associated with the chain:

$$P_{x\to y} = \mathbb{1}_{x\sim y} \frac{1}{n} \frac{\nu(y)}{\nu(x)+\nu(y)}$$

$$\mathcal{E}(\varphi,\varphi) = \sum_{x\sim y} \nu(x) P_{x\to y} |\varphi(x)-\varphi(y)|^2 .$$

Poincaré $\Rightarrow$ Glauber dynamics mixes.

Case I: V quadratic.

example: $V(x) = \frac{\beta}{\sqrt{n}} \sum x_i x_j r_{ij} = \langle x, Ax \rangle$

$r_{ij} \sim N(0,1)$

Sherrigton-kirkpatrick.

Bauerschmidt-Bodineau ('19): $\|A\|_{op} \leq \frac{1}{4}$

$\Rightarrow$ log-Sobolev with $\mathcal{E}'(\varphi,\varphi) = \mathbb{E}_\gamma |\nabla\varphi|^2$.

Thm 1 (Koehler-Zeitouni-E'):

$\|A\|_{op} \leq \frac{1}{4} \Rightarrow$ Poincaré with $\mathcal{E}$.

$$\mathrm{Var}_\gamma[\varphi] \lesssim n\,\mathcal{E}(\varphi,\varphi).$$

Cor: For S-K model in high temp - Glauber dymics mixes in poly time.

Dobrushin condition: $\|\tilde{A}\|_{op} < c$

$\tilde{A}$ is $|A|$ (entrywise).

Def (harmonic extension):

Given $f: \{-1,1\}^n \to \mathbb{R}$

$f = \sum_{A \subset [n]} a_A \prod_{i \in A} x_i$ is the harmonic extension

(as a fn' $[-1,1]^n \to \mathbb{R}$).

Def: ν is C-semi log concave if

$$\nabla^2 \log \frac{d\nu}{d\mu} \preccurlyeq C\,Id.$$

Thm 2: (E'-Shamir)

If ν is C-semi-l-c and φ is 1-Lip

then:

$$\sqrt{\mathrm{Var}_\nu[\varphi]} = o_c(n).$$

A related result by Anari-Gharan-Vinzant:

ν is a measure on $\{0,1\}^n \leftrightarrow 2^{[n]}$.

define:

$$p(x) = \sum_{A \subset [n]} \nu(A) \prod_{i \in A} x_i$$

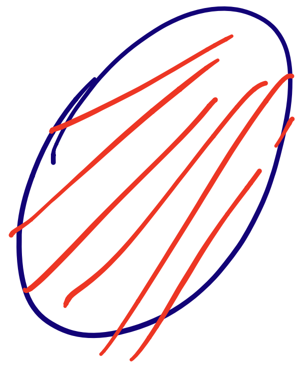
$p(x)$ is log-concave on $\mathbb{R}^n_+ \Rightarrow$ Poincaré.

Needle-decomposition:

Thm: $d\nu = e^{\langle x, Ax\rangle} d\mu$, φ is s.t $\int \varphi \, d\nu = 0$.

$\Rightarrow \exists$ a measure m on $\mathbb{R}^n \times \mathbb{R}^n$ s.t:

$$\nu(A) = \int \mu_{w,v}(A) \, dm(w,v)$$



$$\mu_{w,v}(A) = \frac{\int_A e^{w\cdot x + (v\cdot x)^2} d\mu}{\int e^{w\cdot x + (v\cdot x)^2} d\mu}.$$

s.t m-a.s:

(i) $|v| \leq \|A\|_{op}$

(ii) $\int \varphi \, d\mu_{w,v} = 0$.