Foundations of Non-Monotonic Reasoning
Notation
$S \vdash A$ - from a set of premisses $S$ we can derive a conclusion $A$.

Example
$S$:

- All men are mortal $\forall x.\text{man}(x) \Rightarrow \text{mortal}(x)$
- Socrates is a man. $\text{man}(\text{Socrates})$

$A$:

- Socrates is mortal. $\text{mortal}(\text{Socrates})$
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  \item Socrates is mortal. \( \text{mortal}(\text{Socrates}) \)
\end{itemize}

\( \vdash \) – provability (derivability) relation.
Monotonicity:

For any sets of premisses $S$ and $S'$:

$$S \subseteq S' \quad \text{implies} \quad \{ A : S \vdash A \} \subseteq \{ A : S' \vdash A \}.$$ 

(F1) On Saturday evenings John usually visits his club.

(F2) It is Saturday evening.

(C) John is at the club.
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(F1) On Saturday evenings John usually visits his club.

(F2) It is Saturday evening.

(C) John is at the club.

(F3) Yesterday John had a car accident.
Definition

By *non-monotonic reasoning* we understand the drawing of conclusions which may be invalidated in the light of new information. A logical system is called *non-monotonic* iff its provability relation violates the property of monotonicity.
Non-monotonic reasoning has been primarily studied in the context of common-sense reasoning.
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Unlike formal logical reasoning, whose underlying concept is that of *truth*, common-sense inference is grounded on the concept of *rationality*. We cannot expect that our common-sense conclusions will be always true. However, we always demand them to be rational.
Rationality enjoys two specific properties:

- It is *agent-dependent*: different agents may be of different opinions as to what is rational in a given situation.

- It is *purpose-dependent*: the acceptance of a proposition as a rational conclusion depends on the purpose it is to be used for. For instance, given cursory evidence, I may well assume that Bill is honest and lend him 100 crowns. But I will be more cautious, and try to conduct an investigation, if I was to consider him as my business partner.

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Two important questions:

▶ What really are common-sense conclusions (beliefs)?
▶ What techniques do we use when reaching them?

**Definition (Perlis)**
A proposition $p$ is believed by an agent $g$, i.e. $g$ views $p$ as a rational conclusion, if $g$ is prepared to use $p$ as if it were true.
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A proposition $p$ is believed by an agent $g$, i.e. $g$ views $p$ as a rational conclusion, if $g$ is prepared to use $p$ as if it were true.
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Is (C) based on the rule

(R) From (F1) and (F2) infer (C)?
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Is (C) based on the rule

(R) From (F1) and (F2) infer (C)?

No, it is grounded on the rule

(R') From (F1) and (F2), in the absence of evidence to suspect otherwise, infer (C).
Definition
By a non-monotonic inference pattern (a non-monotonic rule) we understand the following reasoning schema:

“Given information A, in the absence of evidence B, infer a conclusion C.”

An important case:

“Given information A, in the absence of evidence to the contrary, infer C.”
Non-monotonic inference involves two problems:

**Problem of formalization:** To determine how our knowledge is to be formalized. This includes both specifying how this knowledge is to be represented and defining the set of conclusions (beliefs) derivable from such a representation.
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**Belief-revision problem:** To update the current model of the world when new evidence invalidates some currently accepted beliefs.
Non-monotonic reasoning and reasoning about action

Frame problem: The problem of representing which properties persist and which properties change when actions are performed.  
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Qualification problem: How to represent all qualifications which should be satisfied to assure the success of the performed action. Corresponding non-monotonic rule: “In the absence of evidence to the contrary, assume that an action succeeds.”
Typology of non-monotonic reasoning

One should distinguish between *incomplete information* and *incomplete representation* of complete information.

(1) “In the absence of evidence to the contrary, assume that a bird can fly.”

(2) “Unless your name is on a list of winners, assume that you are a loser.”

The list of winners consists of Tom and Bill ⇒ John is a looser.

The list of winners consists of Tom, Bill and John \( \not\Rightarrow \) John is a looser.
Two kinds of non-monotonic reasoning

- **Default reasoning**: The drawing of a rational conclusion, from less than conclusive information, in the absence of evidence making this inference implausible. Default reasoning is defeasible: Any conclusion derived by default can be invalidated by providing new evidence.
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► **Default reasoning**: The drawing of a rational conclusion, from less than conclusive information, in the absence of evidence making this inference implausible. Default reasoning is defeasible: Any conclusion derived by default can be invalidated by providing new evidence.

► **Autoepistemic reasoning**: The drawing of a conclusion, from incomplete representation of complete information, under the assumption that since our information is complete, we would know if the conclusion were false. Purely autoepistemic reasoning is not defeasible; its conclusions cannot be invalidated by new evidence.