Conference on Algebraic Topology

ABSTRACTS

Kraków, June 26 - July 1, 2005

David Blanc
Comparing homotopy theories.

Consider these questions in homotopy theory:

1) How many distinct loop space structures, if any, can a given topological space carry?
2) How can we distinguish between different spaces with equivalent chain complexes?
3) When do two topological spaces have the same rational homotopy type?
4) Can a given graded group G_* be realized as the the homotopy groups $\pi_* X$ of a space X and if so, in how many ways?

Our goal is to describe a unified approach to such problems, in the context of model categories, and show how small variations in the setting can yield different sorts of answers.

Bill Bogley
Metabelian group actions on cubical complexes.

Jens Harlander and I have shown that every finitely generated metabelian group can be embedded in one that is "n-tame" as in the geometric Sigma theory of Robert Bieri and Ralph Strebel. Reporting on continuing work with Harlander, this talk considers actions on contractible cubical complexes that in some cases can be used to verify a conjectural link between n-tameness and the homological finiteness condition FP_n for groups.

Wojciech Chachólski
Representations of spaces

I will present some, not so commonly known, properties of homotopy colimits. I will use them to illustrate how to construct actions of spaces and higher homotopy structures. In particular I will show how to build mapping spaces using resolutions rather than through the "point-wise construction" of hammock localizations.

Marcin Chałupnik

Koszul duality and extensions of polynomial functors.

Introducing of strict polynomial functors by Friedlander and Suslin provided a very convenient language for studying homological problems in the category of $GL_n(\mathbf{k})$–modules. In particular, it enables to perform effective calculations of the Ext–groups between various important $GL_n(\mathbf{k})$–modules. I shall briefly discuss these ideas in the first part of my talk. Then, I shall focus on my recent work on the Koszul duality in the category of strict polynomial functors. This work besides its computational applications also offers some insight into the structure of the derived category of the category of strict polynomial functors.

Mohamad Ebrahimi

Preholomorphic sections on complex manifolds.

In this paper we introduce a hopf algebra structure on a complex manifold and then we give a topology on the set of all preholomorphic sections.at the end we commpute some cohomology groups of holomorphic sections on manifold.

Eric M. Friedlander

Semi-topological K-theory.

We present an overview of this theory developed with Mark Walker which for complex varieties interpolates between algebraic K-theory and topological K-theory. This theory is closely related to morphic cohomology (dual to Lawson homology). Weight filtrations and spectral sequences introduced with Christian Haesemeyer and Mark Walker enable some computations. We shall also discuss certain integral conjectures for this theory which are refinements of familiar conjectures for mod-n algebraic K-theory.

Marek Golasiński

Gottlieb groups of sphers.

(joint work with Juno Mukai)

First, we reintroduce so-called torus homotopy groups by R. Fox, *Homotopy groups and torus homotopy groups*, Ann. Math. **49** (1948), 471-510.

The generalized Gottlieb group is shown to be central in this generalized Fox group. Then, we take up the systematic study of the Gottlieb groups $G_{n+k}(\mathbb{S}^n)$ of spheres $\mathbb{S}^n$ for $k \leq 13$ by means of the classical homotopy theory methods. We fully determine the groups $G_{n+k}(\mathbb{S}^n)$ for $k \leq 13$ except for the 2-primary components in the cases: $k = 9, n = 53; k = 11, n = 115$. We also show that $[\iota_n, \eta_n^2\sigma_{n+2}] = 0$ if $n = 2^i - 7$ for $i \geq 4$.

2000 *Mathematics Subject Classification.* Primary 55Q05, 55Q15, 55Q91; Secondary 55M20. *Key words and phrases*: Gottlieb groups, homotopy groups, torus homotopy groups, Whitehead product.

Jan Gorski

Geometricity of Derived Grassmanian.

Derived algebraic geometry is an extension of algebraic geometry whose main objects of study are generalizations of schemes called D-schemes (or more generally D-stacks). It provides a geometric setting well suited for the construction and study of moduli problems in which the infinitesimal theory (e.g. obstruction theory) and virtual constructions (e.g. virtual fundamental classes) are natural.

A natural question of derived algebraic geometry is to then to construct derived extension of classical moduli spaces (classification spaces). A basic moduli space is the linear Grassmanian $Grass^A(M)$, classifying all the sub-A-modules of a fixed A-module M where A is a finite dimensional algebra. When A is trivial $Grass^A(M)$ is the usual Grassmanian of sub-spaces of the vector space M.

In general $Grass^A(M)$ has an interesting geometry and can be used for instance to construct other classification spaces. In this talk we will show how to construct a derived version $RGrass^A(M)$, which is an important step towards the construction of a derived version of the Quot scheme.

A construction of $RGrass^A(M)$ has previously been done by Kapranov-Ciocan-Fontanine using the theory of dg-schemes. However, the theory of dg-schemes is not suited for the functorial point of view, and therefore this construction is not a solution to a moduli problem. On the contrary our construction is a solution to a natural moduli problem.

Bogusław Hajduk

Diffeomorphisms and almost complex structures on tori

Let $\mathfrak{G}_n$ denote the group of isotopy classes of diffeomorphisms of the torus $\mathbb{T}^n$ supported in a disk. We investigate the action of $\mathfrak{G}_{8k}$ on the set of homotopy clsses of almost complex structures on $\mathbb{T}^{8k}$. Our main theorem says that the action has no fixed points. The proof uses homotopy calculations related to the mapping tori of such diffeomorphisms (which are homeomorphic to $\mathbb{T}^{8k}$) and the index theory of families of Dirac operators. As corollaries we give some partial answers to the following questions:

1. Is any symplectomorphism of $\mathbb{T}^{2k}$ acting trivially on homology isotopic (smoothly) to the identity (McDuff, Salamon 1995)?

2. Does there exist a symplectic form on a torus with an exotic differential structure (Benson, Gordon 1988)?

We show that there exist diffeomorphisms of $\mathbb{T}^{8k}$ supported in a disk which are not isotopic to any symplectomorphism (with respect to any symplectic structure on $\mathbb{T}^{8k}$). This is exactly the part of 1. which can be detected using almost complex structures. For exotic tori obtained as mapping tori T_f of such diffeomorphisms, we get that there is no symplectic form on T_f compatible with the bundle structure of the mapping torus. In particular, the construction of a symplectic structure on T_f by bundle constructions (symplectic connection, Thurston construction) is not possible for such $f's$.

Some progress towards extension of these results using $J-$holomorphic curves will be also discussed. This is a joint work with Aleksy Tralle (cf. Diffeomorphisms and almost complex structures, ArXive: math.DG/0311412).

Bernhard Hanke

Positive scalar curvature with symmetry.

We carry over to an equivariant setting the bordism techniques of Gromov-Lawson and Schoen-Yau for constructing Riemannian metrics of positive scalar curvature. For the groups $G = S^1$ or $G = (Z/p)^r$ the structure of equivariant bordism groups is applied in order to construct

G-invariant metrics of positive scalar curvature on large classes of G-manifolds.

Seoung Ho Lee and Moo Ha Woo

A study on Reidemeister orbit sets.

The Reidemeister orbit set plays a crucial role in the Nielsen type theory of periodic orbits, much as the Reidemeister set does in Nielsen fixed point theory. Extending Ferrario's work on Reidemeister sets, we obtain algebraic results such as addition formulae for Reidemeister orbit sets. Similar formulae for Nielsen type essential orbit numbers are also proved for fibre preserving maps. Also, extending Cardona and Wong's work on relative Reidemeister numbers, we show that the Reidemeister orbit numbers can be used to calculate the relative essential orbit numbers.

Sergey Maksymenko

Homotopy types of stabilizers and orbits of Morse functions on surfaces.

Let M be a smooth compact surface (oriented or not, with boundary or without it) and P either $\mathbb{R}$ or S^1. The group $\mathcal{D}(M)$ of diffeomorphisms of M naturally acts on $C^\infty(M,P)$ by the following rule: if $h \in \mathcal{D}(M)$ and $f \in C^\infty(M,P)$, then $h \cdot f = f \circ h$.

For $f \in C^\infty(M,P)$ let $\mathcal{S}(f) = \{h \in \mathcal{D}(M) \,|\, f \circ h = f\}$ be the *stabilizer* and $\mathcal{O}(f) = \{f \circ h \,|\, h \in \mathcal{D}(M)\}$ be the *orbit* of f under this action.

Let Σ_f be the set of critical points of f and $\mathcal{D}(M,\Sigma_f)$ the group of diffeomorphisms h of M such that $h(\Sigma_f) = \Sigma_f$. Then the stabilizer $\mathcal{S}(f,\Sigma_f)$ and the orbit $\mathcal{O}(f,\Sigma_f)$ of f under the restriction of the above action to $\mathcal{D}(M,\Sigma_f)$ are well defined. Evidently, $\mathcal{S}(f) \subset \mathcal{D}(M,\Sigma_f)$, whence $\mathcal{S}(f,\Sigma_f) = \mathcal{S}(f)$.

Let $\mathcal{S}_{\mathrm{id}}(f)$ be the identity path-component of $\mathcal{S}(f)$ and $\mathcal{O}_f(f)$ and $\mathcal{O}_f(f,\Sigma_f)$ the connected components of $\mathcal{O}(f)$ and $\mathcal{O}(f,\Sigma_f)$ (resp.) in the corresponding compact-open topologies. We endow $\mathcal{S}_{\mathrm{id}}(f)$, $\mathcal{O}_f(f)$, and $\mathcal{O}_f(f,\Sigma_f)$ with C^∞-topologies.

A function $f : M \to P$ will be called *Morse* if $\Sigma_f \subset \mathrm{Int}M$, all critical points of f are non-degenerated, and f is constant on the connected components of ∂M. A Morse mapping f is *generic* if every level-set of f

contains at most one critical point; f is *simple* if every critical component of a level-set of f contains precisely one critical point. Evidently, every generic Morse mapping is simple.

Let $f : M \to P$ be a Morse mapping. We fix some orientation of P. Then the index of a non-degenerated critical point of f is well-defined. Denote by c_i, $(i = 0, 1, 2)$ the number of critical points of f of index i.

Theorem 0.1. *If either $c_1 > 0$ or M is non-orientable, then $\mathcal{S}_{\mathrm{id}}(f)$ is contractible. Otherwise, $\mathcal{S}_{\mathrm{id}}(f)$ is homotopy equivalent to S^1.*

Theorem 0.2. *Suppose that $c_1 > 0$. Then*

(1) *$\mathcal{O}_f(f, \Sigma_f)$ is contractible;*

(2) *$\pi_i \mathcal{O}_f(f) \approx \pi_i M$ for $i \geq 3$ and $\pi_2 \mathcal{O}_f(f) = 0$. In particular, $\mathcal{O}_f(f)$ is aspherical provided so is M. Moreover, $\pi_1 \mathcal{O}_f(f)$ is included in the following exact sequence*

$$0 \to \pi_1 \mathcal{D}(M) \oplus \mathbb{Z}^k \to \pi_1 \mathcal{O}_f(f) \to G \to 0, \tag{0.3}$$

where G is a finite group and $k \geq 0$. If f is simple, then for the surfaces presented in Table 1, the number k is determined only by the number of critical points of f.

Table 1

M	k
$M = S^2$, D^2, $S^1 \times I$, T^2, P^2 with holes	$c_1 - 1$
M is orientable and differs from the surfaces above	$c_0 + c_2$

(3) *Suppose that f is generic. Then the group G in Eq. (0.3) is trivial, whence $\pi_1 \mathcal{O}(f) \approx \pi_1 \mathcal{D}(M) \oplus \mathbb{Z}^k$. In particular, $\pi_1 \mathcal{O}(f)$ is abelian. The homotopy type of $\mathcal{O}_f(f)$ is given in Table 2.*

Table 2

Surface M	*Homotopy type of $\mathcal{O}_f(f)$*
S^2, P^2	$SO(3) \times (S^1)^{c_1 - 1}$
D^2, $S^1 \times I$, *Möbius band Mo*	$(S^1)^{c_1}$
T^2	$(S^1)^{c_1 + 1}$
Klein bottle K	$(S^1)^{k+1}$
other cases	$(S^1)^k$

Theorem 0.4. *If $c_1 = 0$, then f can be represented in the following form*

$$f = p \circ \widetilde{f} : M \xrightarrow{\widetilde{f}} \widetilde{P} \xrightarrow{p} P,$$

where $\widetilde{f}$ is one of the mappings shown in Table 3, $\widetilde{P}$ is either $\mathbb{R}$ or S^1, and p is either a covering map or a diffeomorphism. In this case the homotopy types of $\mathcal{O}_f(f)$ and $\mathcal{O}_f(f, \Sigma_f)$ depend only on $\widetilde{f}$ and are given in Table 3.

TABLE 3

Type	$\widetilde{f} : M \to \widetilde{P}$		c_0	c_1	c_2	$\mathcal{O}_f(f)$	$\mathcal{O}_f(f, \Sigma_f)$
(A)	$S^2 \to \mathbb{R}$	$\widetilde{f}(x, y, z) = z$	1	0	1	S^2	$*$
(B)	$D^2 \to \mathbb{R}$	$\widetilde{f}(x, y) = x^2 + y^2$	1/0	0	0/1	$*$	
(C)	$S^1 \times I \to \mathbb{R}$	$\widetilde{f}(\phi, t) = t$	0	0	0	$*$	
(D)	$T^2 \to S^1$	$\widetilde{f}(x, y) = x,$	0	0	0	S^1	
(E)	$K \to S^1$	$\widetilde{f}(\{x\}, \{y\}) = \{2x\}$	0	0	0	S^1	

Here the Klein bottle K is represented as the factor space of the 2-torus $T^2 \approx S^1 \times S^1$ by the involution $(x, y) \mapsto (x + \pi/2, -y)$ and $*$ means contractibility.

Wacław Marzantowicz

Homotopy methods in periodic points theory.

We present a survey of recent results on the topological periodic points theory. Since the used tools are adaptations of the classical Lefschetz and Nielsen numbers of a map, all they are invariants of the homotopy class of a self-map. An analysis of these invariants gave the Wecken theorem for periodic points, a description of the set of homotopy minimal periods for maps of compact nilmanifolds, also other larger classes of solvmanifolds, and real projective spaces. A complete description of the sets of homotopy minimal periods of a map of three-dimensional solvmanifolds led to the Šarkovskii type theorems. An adaptation of this approach allows us to show that a C^0-map of sphere with unbounded Lefschetz number of its iterations has infinitely many periodic points provided it commutes with a free homeomorphism of finite order. A similar considerations let us to confirm the entropy conjecture for a continuous map of a nilmanifold.

Tomasz Maszczyk

On a pairing between super Lie-Rinehart and periodic cyclic homology.

We consider a pairing producing various cyclic Hochschild cocycles, which led Alain Connes to cyclic cohomology. We are interested in geometrical meaning and homological properties of this pairing. We define a non-trivial pairing between the homology of a Lie-Rinehart (super-)algebra with coefficients in some partial traces and relative periodic cyclic homology. This pairing generalizes the index formula for summable Fredholm modules, the Connes-Kubo formula for the Hall conductivity and the formula computing the K^0-group of a smooth noncommutative torus. It also produces n ew homological invariants of maps contracting orbits of a connected Lie group filling some closed submanifold. Finally we compare it with the characteristic map for the Hopf-cyclic cohomology.

Masaharu Morimoto

G-surgery theory obtaining homology equivalences and the G-fixed point sets of smooth G-actions on shperes.

Let $R = \mathbb{Z}_{(0)}$ $(:= \mathbb{Z})$ or $R = \mathbb{Z}_{(p)}$ such that p is a prime. Let Y be a connected, oriented, smooth manifold of even dimension $n = 2k \geqq 6$ with fundamental group $\pi = \pi_1(Y)$. Cappell-Shaneson introduced a notion of λ-form over a ring homomorphism $\mathbb{Z}[\pi] \to R$, $\lambda = (-1)^k$, in order to develop a surgery theory determining whether a degree-one framed map $f : X \to Y$ is framed cobordant to a homology equivalence $f' : X' \to Y$. They constructed a group $\Gamma^h(\mathbb{Z}[\pi] \to R)$ to which a surgery obstruction $\sigma(f)$ belongs as the set of equivalence classes of their λ-forms.

Let G be a finite group. Let X and Y be equipped with smooth G-actions such that $\dim X^g \leqq k-2$ for all $g \in G \smallsetminus \{1\}$ and $Y^G \neq \emptyset$. Under natural hypotheses on a degree-one G-framed map $f : X \to Y$, we develop an obstruction theory to get a homology equivalence by G-surgery on f. Our G-surgery obstruction $\sigma(f)$ lies in the group $\Gamma^h(\mathbb{Z}[\widetilde{G}] \to R[G])$, where $\widetilde{G} = \pi \rtimes G$. Let $p = 0$ or a prime. We show that if π is finite and the order $|\pi|$ is prime to p then the canonical homomorphism $\Gamma^h(\mathbb{Z}[\widetilde{G}] \to \mathbb{Z}_{(p)}[G]) \to L^h_n(\mathbb{Z}_{(p)}[G])$ is an isomorphism. In the case, the bifunctor $H \mapsto \Gamma^h(\mathbb{Z}[\widetilde{H}] \to \mathbb{Z}_{(p)}[H])$ is a w-Mackey functor, furthermore the Burnside ring $\Omega(G)$ as well as the Grothedieck-Witt ring $\mathrm{GW}_0(\mathbb{Z}, G)$ act on the group $\Gamma^h(\mathbb{Z}[\widetilde{G}] \to \mathbb{Z}_{(p)}[G])$.

Suppose Y is acyclic and the canonical homomorphism $\pi_1(\partial Y) \to \pi_1(Y)$ is bijective. Suppose $\partial X = \partial Y$ and the restriction $\partial f : \partial X \to \partial Y$ is the identity map. Then we obtain $\Delta(f) = id_Y \cup_\partial f : Y \cup_\partial X \to Y$. If f fulfills a certain hypothesis, we can construct a connected sum $f \# \Delta(f) : X \# \Delta(X) \to Y$. One may immediately expect $\sigma(f \# \Delta(f)) = 2\sigma(f)$. But it is not obvious whether the equality really holds, since $\pi_1(X) \neq \pi_1(X \# \Delta(X))$ in general. We prove this equality, moreover show a general formula $\sigma(f \#_G (G \times_H \mathrm{Res}^G_H \Delta(f))) = \sigma(f) + \mathrm{Ind}^{\widetilde{G},G}_{\widetilde{H},H}(\mathrm{Res}^G_H f)$.

Suppose G is a finite abelian Oliver group or a finite perfect group. Let M be a closed smooth manifold. If

(C) each connected component of M is simply connected or stably parallelizable

then the previous work [2] gives a necessary-sufficient condition that M can be realizable as the G-fixed point set of a smooth G-action S on a sphere such that $S^G \neq S^P$ for each Sylow subgroup P of G. We apply our G-surgery theory to improving this result to a complete version, namely one without assuming (C).

References

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Tomasz Mrowka

Progress and Prospects in Floer Homology for 3-manifolds.

In the past few years there has been a burst of activity centering on Floer homology invariants for three manifold. I will discuss some of the structural aspects of Floer homology we have learned in the past few years. Thanks to a surgery long exact sequence these Floer theories are particularly effective in dealing with cobordism questions. Some of the applications I will discuss are the proof of Bing's property P conjecture by Kronheimer and the speaker and the proof of Gordon's conjecture that only the unknot can yield projective space after surgery by Kronheimer, Ozsvath, Szabo and the speaker.

Michele Mulazzani
Representations of $(1,1)$-knots.

A knot K in a 3-manifold N^3 is called a $(1,1)$-knot if there exists a Heegaard splitting of genus one $(N^3, K) = (H, A) \cup_\psi (H', A')$, where H and H' are solid tori, $A \subset H$ and $A' \subset H'$ are properly embedded trivial arcs and $\psi : (\partial H', \partial A') \to (\partial H, \partial A)$ is an attaching homeomorphism. Obviously, N^3 turns out to be a lens space (possibly S^3). In particular, the family of $(1,1)$- knots contains all torus knots and all two-bridge knots in S^3. The topological properties of $(1,1)$-knots have recently been studied in several papers from dierent points of view (see references in [2]).

We develop two different representations of $(1,1)$-knots and study the connections between them.

The first is algebraic: every $(1,1)$-knot is represented by an element of the pure mapping class group of the twice punctured torus $PMCG_2(T)$, where $T = \partial H$ (see [1, 2]). Moreover, there is a surjective map from the kernel of the natural homomorphism $\Omega : PMCG_2(T) \to MCG(T) \simeq SL(2;\mathbb{Z})$, which is a free group of rank two, to the class of all $(1,1)$-knots in a fixed lens space. The second is parametric: using the results of [4] and [3], every $(1,1)$-knot can be represented by a 4-tuple (a, b, c, r) of integer parameters. The above representations are explicitly obtained in many interesting cases, including two-bridge knots and torus knots.

References

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Patrice Ntumba
On the way to Froelicher Lie groups.

In this talk we start by showing that one can obtain by way of diffeomorphisms some classical smooth curves and surfaces. We then introduce the notion of G-bundles in the category of Frölicher spaces. We also review tangent and cotangent bundles of an arbitrary Frölicher space X and prove that these notions coincide with the usual ones when X is a smooth manifold. Finally we study left invariant vector fields on Frölicher Lie groups.

Bob Oliver

Homotopy equivalences of p-completed classifying spaces of finite groups.

In 1996, John Martino and Stewart Priddy conjectured that for any prime p, the p-completions of the classifying spaces of two finite groups G and G' are homotopy equivalent if and only if there is an isomorphism between their Sylow p-subgroups which preserves fusion. Since then, we have proven this conjecture, with a proof which uses the classification theorem for finite simple groups. We will discuss the background to this conjecture and some of its consequences, and also give some indications of the obstruction theory which lies behind the proof.

Petar Pavesic

Homotopy and homology distance.

Spaces X and Y are said to by *homotopy* (resp. *homology distant* if every map self-map of X which factors through Y (i.e. is of the form $X \to Y \to X$) induces nilpotent endomorphisms on all homotopy (resp. homolog) groups of X. The concept of homotopy distance arises naturally in the study of self-maps but it is hard to verify in general. However, under fairly general conditions the concepts of homotopy and homology distance are equivalent. In the talk we will describe some general techniques that can be used to determine whether two spaces are homology distant, and relate these results to product decompositions of spaces into atomic factors.

Carlos Prieto

A classification of cohomology transfers for ramified coverings. (joint work with M. Aguilar)

We construct a cohomology transfer for finite ramified covering maps. Then we define a very general concept of transfer for ramified covering maps and prove a classification theorem for these transfers. This generalizes Roush's classification of transfers for ordinary covering maps. We characterize those representable contravariant functors which admit a family of transfers that have two naturality properties, as well as normalization and stability. This is analogous to Roush's characterization theorem for the case of ordinary covering maps. Finally, we classify these families of transfers and construct some examples.

Holger Reich

On the Farrell-Jones Conjecture for higher algebraic K-Theory. (joint work with A. Bartels)

The Farrell-Jones Conjecture predicts that the algebraic K-Theory of a group ring RG can be expressed in terms of the algebraic K-Theory of the coefficient ring R and homological information about the group. After an introduction to this circle of ideas the talk will report on recent joint work with A. Bartels which builds up on earlier joint work with A. Bartels, T. Farrell and L. Jones. We prove that the Farrell-Jones Conjecture holds in the case where the group is the fundamental group of a closed Riemannian manifold with strictly negative sectional curvature. The result holds for all of K-Theory, in particular for higher K-Theory, and for arbitrary coefficient rings R.

Marco Schlichting

Higher Grothendieck-Witt groups of schemes and derived categories.

In the late 1950s, Raoul Bott proved what is known as "Bott periodicity". In the 1970/80s, Max Karoubi proved an algebraic version of this theorem, now called "Karoubi periodicity". In its current form, the proof of Karoubi periodicity relies on Bott periodicity, and it is only valid under the additional assumption of "2 being invertible".

In my talk I will explain a generalization from "algebraic K-theory" to "hermitian K-theory" of a local-to-global principle of Thomason. This generalization does not rely on Bott-periodicity, yet it gives a new proof of Karoubi- and Bott- periodicity. The work to be presented does not explicitly use the assumption of "2 being invertible"; however, it uses the existence of the correct hermitian K-theory model which is not yet established when "2 is not invertible".

Christian Schlichtkrull
Cyclic algebraic K-theory and the cyclotomic trace.

The definition of cyclic algebraic K-theory is similar to the definition of algebraic K-theory, except that instead of considering the usual classifying space of a category one uses Waldhausen's cyclic version. We shall analyze the cyclic algebraic K-theory in various cases and show how it relates to topological cyclic homology.

Jan Spaliński
Stratified model categories.

The fourth axiom of a model category states that given a commutative square of maps

$$\begin{array}{ccc} A & \xrightarrow{f} & X \\ \Big\downarrow{\scriptstyle i} & & \Big\downarrow{\scriptstyle p} \\ B & \xrightarrow{g} & Y \end{array}$$

with i a cofibration, p a fibration, and either i or p a weak equivalence, a lifting exists in the diagram above. It turns out that for many model categories the two conditions that either i or p above is a weak equivalence can be embedded in an infinite number of conditions which imply the existance of a lifting (roughly, the weak equivalence condition can be split between i and p). The model categories for which this happens are called "stratified" and they are the subject of this talk.

Andras Szücs
Postnikov-like tower for the cobordism of singular maps

a) Cobordism group of embeddings and immersions in unoriented and oriented cases
b) Cobordism of Morin maps
c) Bordism of fold maps
d) The oriented cobordism of fold maps
e) The E^1_{**} member of the spectral sequence.
f) The rational homotopy type in the complex case.
g) Kazarian conjecture: Relationship between Kazarian's classifying space and our ones.

h) Obstructions to elimination of singularities
i) The cohomology theories associated to classifying spaces, and cohomology operations.
j) Coalgebra structures and product formulae

Charles B. Thomas
Calculating the cohomology of the Monster

If $p > 7$ and p divides the order of the Monster simple group, the p-primary part of $H^*(M, \mathbb{Z})$ can be calculated using results already in the literature. I propose to survey these results and relate them to the phenomenon of "moonshine". I hope also to provide some information about what happens at the prime 7, and perhaps consider the wreath product of M and $\mathbb{Z}/2$ ($Y_5 55$), which is of Coxeter type.

Bertrand Toën
Derived Algebraic Geometry.

Derived algebraic geometry is a generalization of algebraic geometry whose purpose is to provide a good setting for representing moduli problems. This generalization is two fold. On one hand derived algebraic geometry deals with (higher) algebraic stacks, a homotopical generalization of the notion of schemes which permits the representability of several moduli problems that could not be represented by classical objects (schemes, algebraic spaces ...) because of the presence of higher symmetries. On the other hand, derived algebraic geometry provides a context where geometric objects possess non-classical points with values in simplicial commutative rings, making deformation theory much better behaved and formalizing the main ideas of the derived deformation theory program. The main purpose of this talk, is to present this theory from a rather informal point of view, emphasising examples as well as the interactions between algebraic geometry and algebraic topology.

Burt Totaro
Grothendieck's theorem on Lie groups and algebraic groups.

Grothendieck proved a precise connection between the torsion in the cohomology of a compact Lie group and the complexity of the corresponding algebraic groups over arbitrary fields. Most of the time will be devoted to an improved proof of the theorem which is simple enough to give during the talk. I use my elementary definition of Chow groups of classifying spaces, which became a part of Voevodsky's homotopy theory. I will conclude with recent calculations and applications. Sample application: every algebraic group of type E_8 over any field becomes isomorphic to the standard (Chevalley) group of type E_8 after a field extension of degree dividing 2880. The number 2880 is optimal and is found by a topological calculation.

Angelo Vistoli

Chow rings and cohomology of classifying spaces of classical groups (particularly PGL_p).

Cohomology rings of classifying spaces for algebraic groups have been studied for a long time by topologists. Recently Totaro has introduced an algebraic analogue, the Chow ring of a classifying space; this is typically simpler its cohomology, but still preserves some of its key features.

After a survey of the known results on Chow rings of classifying spaces of classical groups, I will discuss my results on the Chow ring and the cohomology of the classifying space of PGL_p, where p is an odd prime, and their link with the classical problem on the cyclicity of division algebras of prime degree.

Andrzej Weber

Integral equivariant cohomology of algebraic varieties.

Let X be a smooth and compact complex algebraic variety. Suppose an algebraic linear group G acts on X. If G is connected then (for example by Hodge theory) the rational equivariant cohomology of X is a free module over $H^*(BG)$. For integer coefficients the above statement holds only for so called special groups. They can be characterized by the equivalent conditions:

1. $H^*(BT) \to H^*(G/T)$ is surjective,
2. $H^*(BG)$ has no torsion,
3. every algebraic G-bundle is locally trivial in Zariski topology,
4. the maximal semisimple subgroup of G is a product of Sl_n's and Sp_n's.

Also we give a characterization of G in terms of motives. For torus actions we discuss a relations with the localization theorem.

Michael Weiss

Stratified spaces and homotopy colimit decompositions, with applications to surface theory.

A stratified space is a space with a partition into locally closed subsets, subject to some conditions. Stratified spaces occur a lot in differential topology, but the homotopy theoretic implications of a stratification are often ignored. In this talk I want to describe two ways of "decomposing" a stratified space in a homotopy theoretic sense. None of these decompositions is entirely new. One of them is used in joint work of Ib Madsen and myself on surface bundles and the Mumford conjecture. To the extent that time permits, this application to surface bundles will be sketched.

Nobuaki Yagita

Algebraic cobordism of a Pfister quadric (joint work with A.Vishik)

Levine and Morel defined algebraic cobordism theory as a universal theory of cohomology theories having transfers. On the other hand, Voevodsky defined motivic cobordism theory by using spectrum MGL in the stable A1-homotopy category. We explicitly compute these cobordism theories for Pfister quadric and show they are isomorphic for this case.

Min Yan

Periodicity in Equivariant Surgery

(joint work with Shmuel Weinberger)

For topological manifolds, the classical equivariant surgery theory has the four-fold periodicity: $S(M) = S(M \times D^4)$. For homotopically stratified equivariant topological manifolds acted by compact Lie groups, we extend the periodicity to the case the disk is replaced by twice of any complex representation. We also prove the bundled version of the extension, which is some sort of Thom isomorphism theorem for equivariant surgery theory.

Alexey Zhubr

Normal homotopy classification of 6-manifolds.

Consider the class of closed simply connected oriented smooth 6-manifolds. Complete smooth and homotopy classification of these was obtained earlier by the author [1], while the classification in question — up to normal homotopy equivalence — appeared to be more "persistent" and is not yet known in full. At present, we have some sort of "upper bound" and "lower bound" (which give exact answer in some special cases). Note that this type of classification is of interest by several reasons, one of which is that the difference between normal homotopy equivalence and diffeomorphism is precisely what is measured by classical surgery obstructions and cannot be detected by "conventional" invariants.

First we introduce some notation and reformulate some results of [1, 2].

1. Notation and preliminary results. Let M be some fixed manifold of the type above. Denote by $\mathcal{S}(M)$ the set of stable diffeomorphism classes of pairs $\{M', f\}$, where M' is a manifold of the same type, $f : H_2(M') \to H_2(M)$ an isomorphism satisfying the condition $f^* w_2(M) = w_2(M')$, and "stable" means "up to connected sum with copies of $S^3 \times S^3$". Speaking somewhat loosely, one can say that $\mathcal{S}(M)$ is the set of all manifolds having the group H_2 and the class w_2 the same as M. Let G be a finitely generated abelian group, $w : G \to \mathbf{Z}/2$ a homomorphism. Denote by $\Omega_6^{\mathrm{spin}}(G, 2; w)$, or in shorthand by $\Omega(G, w)$, the group of bordism classes of maps $f : M' \to K(G, 2)$ with $f^*(w) = w_2(M')$. Taking $G = H_2(M)$, there is the natural map

$$\mathcal{S}(M) \to \Omega(G, w) \tag{1}$$

In [2] (see also [3]) it is proved that this is a bijection, which provides basis for the classification results of [1] (essentially, what the paper [1] consists of is calculation of the groups $\Omega(G, w)$).

Let μ denote the natural homomorphism $\Omega(G, w) \to H_6(G, 2)$. Let $\mathcal{S}_0(M)$ be the subset of $\mathcal{S}(M)$ defined by $\mu\{M', f\} = \mu\{M, \mathrm{id}\}$, and let $\Omega_0(G, w)$ be the kernel of μ. We have again the bijection $\mathcal{S}_0(M) \to \Omega_0(G, w)$ (taking the difference $\mu\{M', f\} - \mu\{M, \mathrm{id}\}$). Choose now a decomposition

$$G = \mathbf{Z}/2^m \times \mathbf{Z}/2^{a_1} \times \ldots \times \mathbf{Z}/2^{m_1} \times \ldots \times \mathbf{Z}/2^{b_1} \times \ldots \times G_{\mathrm{odd}} \ ,$$

where (a) each factor may be absent; (b) the first factor $\mathbf{Z}/2^m$ is the "support" of the Stiefel-Whitney class w (in the sense that w is zero on all the other factors, and non-zero on this one); (c) any of the numbers

m, a_i, m_i, b_i may be equal to infinity (in particular, we assume $m = \infty$ if $w = 0$); (d) the relations $a_i < m = m_i < b_i$ for all i hold. Then we have the following

Theorem (see [1], subsection 5.33). The group $\Omega_0(G, w)$ is isomorphic to

$$\mathbf{Z}/2^{m-2} \times \mathbf{Z}/2^{a_1-1} \times \ldots \times \mathbf{Z}/2^{m_1-2} \times \ldots \times \mathbf{Z}/2^{b_1-3} \times \ldots \times 3G_{\mathrm{odd}} \quad (2)$$

In fact, due to bijection (1), this gives the "enumeration" of all manifolds M' having $H_2(M')$, $w_2(M')$, and cohomology operations on $H^*(M'; A)$, the same as M. It should be noted that this is again "speaking loosely": to obtain the "honest" classification, we have to further calculate $\Omega_0(G, w)/\mathrm{Aut}(G, w)$, which is generally non-trivial.

Now let $\mathcal{S}^h(M)$ denote the subset of $\mathcal{S}(M)$ composed of those isomorphisms $H_2(M') \to H_2(M')$ induced by homotopy equivalences. In view of bijection (1), we can regard this as a subset of $\Omega_0(G, w)$. Standard (and easy) surgery shows (see [2]) that this coincides with the image of the group $\Omega_6^{\mathrm{spin}}(M; w_2(M))_0$ of bordism classes of maps $f : M' \to M$ with $f^* w_2(M) = w_2(M')$, of degree zero. This group can be explicitly calculated, which provides the homotopy classification (see [1]).

2. Normal homotopy classification: upper bound. Denote by $\mathcal{S}^{nh}(M)$ the subset of $\mathcal{S}(M)$ composed of those isomorphisms $H_2(M') \to H_2(M)$ induced by normal homotopy equivalences. Like above, we can regard this as a subgroup of $\Omega_0(G, w)$.

Consider now the subgroup $\Omega_0^h(G, w) \subset \Omega_0(G, w)$, generated by elements of order 2 in $\mathbf{Z}/2^{m-2}$ and in each $\mathbf{Z}/2^{a_1-1}$ of decomposition (2) (so $\Omega_0^h(G, w)$ is a product of several $\mathbf{Z}_2$'s). It can be shown that this "definition" is in fact invariant (not depending on specific decomposition of G).

Theorem. The subgroup $\mathcal{S}^{nh}(M)$ is contained in the intersection of subgroups $\Omega_0^h(G, w)$ and $\mathcal{S}^{nh}(M)$; moreover, if the decomposition (2) has no finite b_i's, then $\mathcal{S}^{nh}(M)$ is precisely $\Omega_0^h(G, w) \cap \mathcal{S}^{nh}(M)$.

Thus, we have the "upper bound" for the set of manifolds normally homotopy equivalent to a given M; this "upper bound" becomes precise answer if $H_2(M)$ satisfies some condition on 2-torsion (no elements of "too great" 2-primary order). Note that this condition is automatically satisfied if $w_2(M) = 0$ (or, more generally, if $w_2(M)$ is an integral class). Note also that this "upper bound" becomes zero (hence exact) for $m = 1, 2$.

3. Normal homotopy classification: lower bound. If the 2-torsion condition above is not fulfilled, then we can prove only the following weaker result.

Theorem. If $m \geq 3$, then the group $\mathcal{S}^{nh}(M)$ contains the element of order 2 in $\mathbf{Z}/2^{m-2}$ of decomposition (2) (so it is non-trivial).

In fact, the following quite probably is true:

Conjecture. The group $\mathcal{S}^{nh}(M)$ coincides with $\Omega_0^h(G, w) \cap \mathcal{S}^{nh}(M)$.

References

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Krzysztof Ziemiański

Homotopy unitary representations of p-compact groups

p-Compact groups are homotopy-theoretic analogues of compact Lie groups. By a homotopy complex representation we mean a homomorphism into a p-compact unitary group. Homotopy representation theory seems to be distant from a classical algebraic theory. I will provide a classification of homotopy representations of some small p-compact groups and discuss interesting phenomena which appear in this case. Furthermore, I will present a construction of an adjoint representation in the case if p-is odd and an example of a faithful representation of $DI(4)$ at prime 2 – the only example which does not admit an adjoint representation.