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KAROL BORSUK, PERSONAL REMINISCENCES

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Dedicated to the memory of Karol Borsuk

In the academic year 1930-31 I was a first year student at the University of Warsaw, while Karol Borsuk was an assistant conducting exercises in real analysis. I was a member of a class which was huge but he soon started to notice me and we got involved in several conversations. In the spring of 1931 he received his doctorate and I attended the ceremony. At the same time I attended a course on set theory given by Docent Bronislaw Knaster. There were two other students in the course, however, I was the only one who did all the homework. I struck up a friendship with Knaster which lasted as long as he did. Set theory naturally led to Topology, which in Warsaw meant strictly Set-theoretical Topology.

I remember a curious incident. In the fall of 1931 I was browsing through the Mathematics Library and I came across a book entitled Topology by Solomon Lefschetz. I looked at the bibliography to see to what extent the "Polish School" was quoted. I found only one reference. It was a paper of Knaster, Kuratowski and Mazurkiewicz in volume 15 of Fundamenta Matematicae containing a combinatorial proof of the Brouwer Fixed Point Theorem. I was very surprised to find no other references and I conveyed my concern to Dr. Adolf Lindenbaum (an excellent logician) who was then the assistant in charge of the library. He told me that it was a terminological misunderstanding, that the book was not about Topology but about some sort of algebra.

With time Borsuk learned about Algebraic Topology (mostly Vietoris cycles) and as he learned so did I. He went to Zürich, Innsbruck and Vienna in 1932 and when he came back I eagerly learned about what was happening on the "other side". In 1934 Kuratowski moved from Lwów to Warsaw to a Professorship of Mathematics. Since Borsuk was only a Docent, Kuratowski became my official sponsor with Borsuk, my de facto teacher.

Maps of spaces into spheres were at that time a major topic in Borsuk's work. I picked this up and made maps into the circle the topic of my dissertation. This turned out to be a powerfull tool for handling both qualitative and quantitative problems and thus helped bridge some of the gap between algebraic and set-theoretic topology. I presented my dissertation in 1936. Although it was written independently, the general subject was very much inspired by Borsuk's work. The period 1936–1939 was a period in which both Borsuk

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and ~~myself~~ I tried intensively to algebraisize ourselfes and each other. It was a very exciting period for me as papers of Alexander, Brouwer, Hopf and Hurewicz became accesible to me revealing riches beyond belief. Borsuk was constantly by my side as friendly advisor and father confessor.

In 1936 Borsuk and I published a joint paper ("Uber stetige Abbildungen der Teilmenge euklidischer Raume auf die Kreislinie", Fund. Math. 26, 1936). The main problem concerning us was the following: given a solenoid Σ in S^3 how big is the set S of homotopy classes of maps $f : S^3\backslash\Sigma \to S^2$? Our algebraic equipment was so poor that we could not tackle the problem in the whole generality even though all the tools needed were in our paper. In 1938, using the newly developed "obstruction theory", I established that the set S in question is equipotent to the appropriately defined homology group $H^1(S^3\backslash\Sigma, \mathbb{Z})$ (cf. "Cohomology and continuous maps", Ann. of Math., 41, 1940). This was done in Warsaw before my departure to America in the Spring 1939.

independently

At this point the problem was taken up by Norman Steenrod ("Regular Cycles of Compact Metric Spaces", Ann. of Math., 41, 1940). With the aid of "regular cycles" he computed the group $H^1(S^3\backslash\Sigma, \mathbb{Z})$ and as a consequence resolved our problem by showing that the the set S is uncountable.

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When Saunders MacLane lectured in 1941 at the University of Michigan on group extensions one of the group appearing on the blackboard was exactly the group calculated by Steenrod. I recognized it and spoke about it to MacLane. The result was the joint paper: "~~On~~ Group extensions and homology", Ann. of Math. 43, 1942. This was the birth of Homological Algebra.

Correction

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Niech będzie

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