

In Memoriam: Edmund Puczyłowski (13.02.1948 - 19.10.2021)

Tomasz Puczyłowski¹

Edmund was born in a small town in north-eastern Poland. His ancestors were foresters in the old Augustów Forest. After the Second World War, Edmund's father, as a forester, was sent to the Romincka Forest in the former East Prussia. There, in a small forest settlement, in a post-German forester's lodge, Edmund spent his childhood. After finishing the village primary school (he had to walk 4 km one way to get there — and these were areas famous for their long, cold and snowy winters), he went to the Forest Technical School in Białowieża, nearly 300 km away, where his older brother Wiesław was already studying. There, his talent for mathematics was discovered by Professor Zych. It was he who encouraged Edmund (and his other pupils) to take part in the Mathematical Olympiad. Edmund won that Olympiad. As a result, he qualified for the International Mathematical Olympiad (then held in Yugoslavia) and was subsequently admitted, without any entrance exams, to the Faculty of Mathematics at the University of Warsaw. He worked mainly in Warsaw at his home faculty. But he also worked in Białystok, where we lived.

¹Tomasz Puczyłowski is one of Edmund Puczyłowski's sons and a professor of philosophy at the University of Warsaw

In 1975 he got his Ph.D., in 1986 his habilitation, and in 1995 he became a professor of mathematics. When he was 23, he married Maria (née Szczepanek). They had three children — Barbara (1971), Tomasz (1974) and Michał Ludwik (1982). He has two grandchildren — Antoni and Stanisław. As a mathematician, he was extremely hard-working and very talented. He loved doing mathematics. He devoted his free time to it, it was his real hobby. Furthermore, he also devoted a lot of time to the Mathematical Olympiad, which he chaired for several years, and to the Institute of Mathematics at the University of Warsaw, of which he was director until his stroke in 2015. In his busy schedule, between conferences, mathematics, the Institute, the Olympiad, there was also time for us. For Mum, children and grandchildren, Dad was a good man. Gentle towards others, understanding and generous. I remember his conscientiousness, thoroughness and diligence. We all loved him so much, he will remain in our hearts forever.

Mikhail Chebotar²

Edmund Puczyłowski and 'that problem'

²Misha Chebotar is a professor at Kent University, USA. His email address is mchebot2@kent.edu.

Formally, Edmund Puczyłowski and I could have met as early as in 1996 when both of us were attending a very nice Ring Theory conference in Miskolc, Hungary. However, we tried to recall whether we really met each other there or not, and decided that it probably did not happen.

We first met in Taiwan a few years later: Edmund was a frequent visitor of my Ph.D. advisor Kostia Beidar and our Taiwanese friends Yuen Fong, Wen-Fong Ke and Pjek-Hwee Lee, and I was a postdoc at National Taiwan University. By that time I have already been familiar with two recent breakthroughs in Radical Theory: Edmund's joint paper with Agata Smoktunowicz on the Brown-McCoy radical of polynomial rings, and Agata's famous solution of Amitsur's problem on whether the polynomial ring over a nil ring should be nil. Edmund was working with Kostia and Yuen on another important paper: they proved that polynomial ring over a nil ring is Behrens radical, and I just started my journey in Radical Theory by reading some papers. The problem that puzzled me the most was whether a polynomial ring in several variables over a nil ring is Brown-McCoy radical. For non-commuting variables this problem was posed by Edmund in 1993, and for commuting variables it was posed by Edmund and Agata in 1998 (it turned out that commuting and noncommuting cases are essentially equivalent due to results obtained independently by Agata, and Miguel Ferrero and Robert Wisbauer). I liked this problem, because it could be restated in terms of the central closure of a prime ring (the result due to Miguel and Robert) and my previous backround in prime rings could be of some use.

As I learned later, this problem was of a significant interest for most of mathematicians working in Radical Theory: everyone believed in the positive answer, but nobody could prove it until recently. Our team (Edmund, Pjek-Hwee, Wen-Fong and I) spent over 12 years working together on this problem. We were meeting many times in the US

and in Taiwan, we were getting some partial results, we were working on some other problems as well (for example, in our paper on commutators and nilpotent elements in simple rings we solved 3 open problems posed by Hesteyn in 1961). There were some funny stories related to our cooperation and meetings. Let me tell you one of them. Once in Taipei I noticed that Edmund was wearing many different T-shirts with the National Taiwan University Logo. I knew that he liked NTU, but I did not expect him to like it that much, so I asked Edmund a natural question. He answered that Lufthansa lost his luggage and he needed some clothes, so he simply went to a nearest store that was in the NTU campus.

In 2011 we had an algebra conference in Kent, Ohio, and Edmund, Wen-Fong and Lenny Makar-Limanov were staying in my house. After a dinner we had a conversation about what problems we want to solve before our retirement. Lenny asked me if it is the Koethe Conjecture that I want to solve, and my answer surprised him, since I mentioned 'that problem'. Then Lenny said that for polynomials in several variables there are too many ways to define 'degrees', so algebraic geometers prefer to work with Newton polytopes in order to have some geometric intuition behind the object. The next morning Edmund told me that he thought about Lenny's words and the Newton polytopes maybe the way to solve 'that problem', but of course we could not immediately find how to do it. We did it 5 years later, indeed, using Newton polytopes, central closure, methods of combinatorial algebra and basic convex geometry, and this was my last joint paper with Edmund (M. Chebotar, W.-F. Ke, P.-H. Lee and E.R. Puczyłowski. On polynomial rings over nil rings in several variables and the central closure of prime nil rings). Normally, one should be happy after such a long journey, but three of us were pretty sad. The Portugal national football team probably had such a feeling after winning Euro 2016. Edmund was a big football

fan, we watched many games together, so he would definitely understand what I wanted to say here. We will miss him dearly.

Marzena Filipowicz-Chomko³

I met Professor Edmund Puczyłowski in August 1999. That was a moment when our long relationship and our common journey across the ocean of mathematics started. After many years, it turned out that my Ph.D. dissertation was the last that Edmund supervised. It is not easy to summarize how important role he played in my life just in a few sentences.

He was not only my master but also a friend and a life coach. Edmund always spoke about mathematics with great passion. It consumed him for long lonely hours but after that, he liked sharing his findings with others. The most fascinating thing was that I could participate in the whole process of creation. I could see his genius which he never hesitated to share. Another Edmund's trait was that he loved to make plans and he was always well organized. I know for a fact that in his office he kept files of all his Ph.D. students in which he gathered ideas and materials for our future scientific work. Each file had its own title, the one written on mine said: Filial rings, yesterday, today, tomorrow. I still keep it and it reminds me that Edmund always knew my dreams, aspirations, and scientific possibilities better than I had known myself.

Although he loved interacting, inspiring, motivating, and helping other people, he also loved mathematics and it made him a really demanding supervisor. I was never discouraged by this fact, on the contrary, he demanded much more from himself, so motivated me to work even harder. Each article was a result of hours and hours of discussions and many pages written with his careful handwriting. It was a constant search for even

more precise and elegant proof. At times he got on my nerves as he would correct a single sentence all over again. It just had to be perfect.

Edmund had also a phenomenal ability to connect facts. His brain and perspective were extremely broad and he tried to tackle each problem from various angles. He would never give up. Scientific career and academic work always have ups and downs, and each of us has moments of doubt, but I hope in the future I will be able to tell myself the same what Edmund once said: *“I have never regretted choosing mathematics. There is everything in it, from a penny for bread to poetry.”*

Eric Jespers⁴

My first research visit as a junior ph.d. student was to the Department of Mathematics of the University of Warsaw in the early 1980's. I was attracted by the work of the algebra research group led by Professor Krempa. The group was located in central Warsaw, in the impressive Pałac Kultury i Nauki. The large research group had, a not so large, office on the 10th floor if I remember well. Most people popped in for mathematical discussions and to exchange ideas. Most of the time people worked at home. It is during that first research visit I met Edmund for the first time.

It was immediate clear that he liked social contact and was keen on exchanging ideas and to learn from each other (even from me as a very junior mathematician). All our conversations were very pleasant and very motivating and he considered everyone at an equal level. But not only his mathematics was exciting, we also talked a lot about daily life. In particular about the situation in Poland and the solidarity movement. For me all this was very new and an eye opener. We got along very well and Edmund took me on visits to his home in

³Marzena Filipowicz-Chomko is a professor at Białystok University of Technology. Her email address is m.filipowicz@pb.edu.pl.

⁴Eric Jespers is a professor emeritus at the Vrije Universiteit Brussel, Belgium. His email address is Eric.Jespers@vub.be

Białystok (a city in the North east of Poland) and to that of his family members in the country side. We had a wonderful time and I learned to know his family. Here I learned, in an extreme way, that one can do mathematics, and being social at the same time, anywhere: on the train, while waiting in line at stores, while walking in the forest, while visiting unknown people and making new friends — something that Edmund clearly was used to.

This first research visit was followed by several research visits to Warsaw and also by visits of Edmund (and Jan Krempa) to Belgium. Edmund and Jan usually stayed in my home. During those visits we realized our first mathematical joint work. Also, Edmund taught me a lot on how to write a mathematical paper. During those visits one discovered even more Edmund's good nature, genuine social interest and social skills. A real gentleman.

In the following years we also met many times at international conferences and workshops. Edmund always gave very clear and exciting lectures on his beautiful results (often related to the Köthe conjecture) and in his presentations he kept the audience in mind. During informal discussions he exchanged many ideas and motivated many people, both junior and senior. He truly was an excellent motivating and stimulating educator and researcher.

Although we met seldom during the past 20 years, we exchanged yearly new years greetings. The excellent mathematician, exciting educator, gentleman and friend Edmund will be truly missed.

Miguel Ferrero⁵

Actually I do not remember how I made contact with one of Edmund's paper for the first time, but I remember that I immediately became interested in some of his results related to radical theory and especially to the

Köthe conjecture, since those were subjects that attracted me from the beginning of my studies. So I wanted to have personal contact with him and for this reason I invited him to visit our university (Universidade Federal do Rio Grande to Sul, Porto Alegre, Brazil) Then he joined us and spent some months here, gave a seminar and helped me to learn more on the subjects he knew so deeply about.

The day he arrived to Porto Alegre was not easy for me. For some reason his plane did not arrived on time. As I knew he had never been to Brazil I was afraid that he could have some difficulties on his arrival. But finally all turned out well and we met.

At the hotel he was nicely surprised, when he met some people that usually played chess there; an activity that he used to practice as well and that brought him good times in his free hours in Porto Alegre.

At the university he started presenting a seminar for our group and soon we began discussing questions of common interest. This was very important since I realised how this interaction would be encouraging for me. So I realised that it had been a very good decision to invite him. He was so kind and open minded, we easily had good scientific discussions and I was very happy with the chance to share my work with him.

Thanks to this opportunity and his influence I got to know new places and researchers, for instance through my visit to the Banach Center in Warsaw afterwards. I have very good memory of my visit to Poland when the country was yet a communist country. In particular one day he invited me to visit his birthtown, Białystok. It was winter time and very cold. One night we were in his house and he suggested to go for a walk in the surrounding. It was -17 Celsius degrees. An unforgettable experience for a south-american like me.

I know many people that have good memories of Edmund. His knowledge of algebra was very huge and I could mention a lot of things concerning this, but I leave this task to other colleagues who can do it better. In

⁵Miguel Ferrero is professor aposentado at the Universidade Federal do Rio Grande to Sul, Porto Alegre, Brazil. His email address is ferreroma@icloud.com

particular the note by Richard Wiegandt is very interesting and complete in this regard.

For his family, my condolences. It will be not easy to continue your life without him, I am very sorry.

Pjek-Hwee Lee Wen-Fong Ke⁶

In our memory, Edmund was always serious in deep thinking. But, when he laughed, we could easily share his joyfulness from his heart.

He came to Taiwan almost every year since 2002, and only stopped after he got the stroke. He usually stayed a week or two in National Taiwan University, Taipei, where he and Pjek-Hwee worked together on some projects, then came to National Cheng Kung University, Tainan, where he and Wen-Fong would have some discussions. He then went back to Taipei again for a week or so before returning back to Warsaw. Besides mathematics and friends, one thing Edmund enjoyed a lot was the variety of foods we have in Taiwan. There were also different offers he could get in Taipei and in Tainan.

We all think that he was the mathematician to have lots of insights on Köthe's problem, and that maybe was the reason for him to appear in deep thinking most of the time. On the other hand, he talked about the problems in the administrative duties he had as the head of the Faculty of Mathematics, Informatics and Mechanics of the University of Warsaw. That required some serious attentions from him. But he was so sincere and willing to help everyone no matter what the situation was.

The stroke affected his ability to speak in English; but Edmund still tried to pick it up again, just it was not easy at all. Wen-Fong went visited him three times (2017, 2018, and

2019). It was sad that the COVID-19 pandemic prevented us from visiting him before he past away. But sure Edmund will always be in our memory.

Ali Majidinya⁷

I'm not in a position to introduce or describe professor Edmund in his mathematical life; I can just remember him as one of the kindest and the best mathematicians in several points of view:

I was lucky to live in Poland with my family in 2010 for a research opportunity and I had been invited by professor Edmund. Just one week prior to leaving for Poland, unfortunately my wife lost her mother and it was a very bad event at that time for all of us. We had to go to another country with a broken heart. Especially, my wife was suffering great sorrow. But when we arrived to Warsaw airport and we visited professor Edmund, in our first visit, I can say most of the sadness went away. When professor Edmund learned about the sad event and he met us with our great sorrow, he said to us: "Don't worry and assume that I'm grandfather to your son ...". Really he was as a good father for me and during the time we were in Poland, he treated us as a good and kind father would have done toward his children. He and his wife visited us almost every weekend.

Edmund didn't spare us help in any situation, living, doing research and even going on picnics. I was able to do my research in Poland in as systematic way as possible. For example, he had organized weekly mathematical seminars which I often attended. I could learn many things, in particular Edmund's presentations and his mathematical discussions and ideas were very helpful and interesting. I remember him as an international professor of mathematics and an epitome of kindness and good humanity. I can say

⁶Pjek-Hwee Lee is a professor emeritus at the National Taiwan University and Wen-Fong Ke is a professor at the National Cheng Kung University in Tainan, Taiwan. His email address is wfke@mail.ncku.edu.tw

⁷Ali Majidinya is an assistant professor at Salman Farsi University of Kazerun, Iran. His email address is ali.majidinya@kazerunsfu.ac.ir

that he was one of the best effective people in my life. In this way his memory lives on with us forever. His soul is happy.

Ryszard Mazurek⁸

I would like to pay tribute to an extraordinary man and mathematician Edmund Puczyłowski, whom I was lucky to meet in 1986 when I was a young academic teacher.

In 1984, I finished my studies at the University of Wrocław with a master's thesis on algebraic number theory and soon joined the Institute of Mathematics of the Białystok Branch of Warsaw University. Early in 1986 after my year-long compulsory military service, I returned to Białystok, but meanwhile my possible supervisor in number theory left the university, and just at that time I met Edmund, a smiling, slim and vigorous algebraist. Edmund was employed at the University of Warsaw, but at that time he was appointed to teach at the branch of the University in Białystok, where he lived. Knowing my situation, Edmund has encouraged me to switch my interest to noncommutative ring theory and enthusiastically created a plan for my work on this new topic. This shows that Edmund was looking for young people to introduce them into his scientific field and to share his deep knowledge with them. Edmund had a specific method of introducing interested young people to ring theory by thoughtfully preparing series of problems, that we were asked to solve on our own, leading us to thorough understanding proofs of important and well-known theorems of the theory. Some people dropped out in the process, while others moved on, reaching the next stage, which consisted of studying specific topics and current problems advised by Edmund.

In my case, the first suggested area of study were chain rings, i.e. rings whose one-sided ideals are totally ordered by set inclusion.

This subject appealed to me very much, and I am about to explain why the class of chain rings caught Edmund's interest. As a prominent specialist in the theory of radicals, he was at that time especially interested in chain domains R with unity possessing only one proper nonzero ideal; this ideal is just the Jacobson radical $J(R)$ of R . Such chain domains R are important in the theory of radicals for the following two reasons: firstly, $J(R)$ is then a simple Jacobson radical domain (the long remaining open problem whether simple Jacobson radical domains do exist was positively solved by E. Sławiński in 1961 by a complicated example, what at that time was a revelation on a global scale) and secondly, if the factor ring $R/J(R)$ is a field of prime order p , then $J(R)$ generates an atom in the lattice of all radicals. Another reason for Edmund's interest in chain rings was his idea to treat the class of chain rings as a testing class for open problems in ring theory; namely, when approaching an open question, he proposed at the beginning to find how will be an answer if we restrict the question to chain rings only. A basis for this idea was the fact that the class of chain rings is appropriately balanced: at the one hand, the rings in this class have a quite regular structure, and at the other, there are subtle examples of rings in this class.

Afterward Edmund suggested me to study distributive rings, i.e. rings whose lattices of one-sided ideals are distributive. This class of rings, which obviously contains all chain rings, was the theme of my doctoral thesis supervised by Edmund and defended at the University of Warsaw in 1993. Edmund was easy approachable, very supportive and friendly, and his instructions were encouraging and inspiring, both when I was entering the theory of noncommutative rings and when was preparing the thesis; and later whenever I needed his advice. Rigorous, but also very helpful to his students preparing doctoral thesis, he demanded precision, clarity, and concision in the writing process. Under his guidance we received a valuable education in preparing

⁸Ryszard Mazurek is a professor at the Białystok University of Technology. His email address is r.mazurek@pb.edu.pl

mathematical texts. He also made us realize that a doctorate is just a natural stage in the career, and it is unacceptable that after defending it a person will be “hanging emaciated on the fence” (as he figuratively said), without any desire to continue research.

Edmund stimulated our continuous cooperation with algebraists from the University of Warsaw. He very often delivered talks at our algebraic seminar in Białystok, and also often brought eminent foreign algebraists to the seminar or for a longer stay in Białystok, and thanks to that we had personal and close contact with them, which sometimes led us to a closer collaboration. He initiated various activities for the benefit of the mathematical community of Białystok.

Such consistent and enthusiastic activities of Edmund created in the course of time a significant group of algebraists in Białystok. Among them were found mainly Edmund’s alumni (in the order of receiving their doctorate degrees: P. Grzeszczuk, R.R. Andruszkiewicz, me, E. Roszkowska, M. Kępczyk, M. Filipowicz-Chomko) and their descendants. All of Edmund’s Białystok students wrote their doctoral theses while regularly involving in teaching and all the theses were successfully defended at the University of Warsaw. I remember Edmund saying that when he decided to study mathematics, he pictured himself in the future as a high school teacher, leading a mathematical circle for schoolkids to awaken their passion for the queen of all sciences. Fortunately for us in Białystok, things turned out slightly differently: Edmund became a distinguished university professor and led the “algebraic circle” for us. It should also be added that he supervised the Ph.D. thesis of the outstanding algebraist Agata Smoktunowicz.

Edmund was a brilliant mathematician and a truly wonderful man. He represents to me the image of a perfect mentor. He will be remembered by all of us, and we are very grateful to have opportunity to know and to collaborate with such an outstanding person.

Patrick F. Smith⁹

It is a great privilege as well as a huge pleasure to be asked to write a few words about my late friend Edmund Puczyłowski. I can recall how our friendship began and the sequence of events almost 40 years ago when we first met. Jan Krempa, Jan Okninski and Edmund came to Glasgow in August 1984 for a short visit and to attend the summer meeting of the Edinburgh Mathematical Society in St Andrews. In December 1984 I returned the visit to the University of Warsaw when the Algebra Department occupied one room on the Thirteenth Floor of the Palace of Science and Culture in Central Warsaw. Jan Okninski and his family very kindly let me stay with them and made me very comfortable. Warsaw then was very different to Warsaw now. The supermarket shelves were not well stocked and the bread shelves had small pieces of paper to be used to test how old the bread was! The heating in the Palace was unpredictable being supplied by a power station miles away and on at least one occasion failed altogether. It was December and none too warm when the heating went off! The Algebra group of about a dozen people shared 5 desks but nobody complained. There were two lifts in the Palace and one day I decided to take the posher one. The woman looking after the lift let fly with a tirade in Polish, none of which I understood, to leave me in no doubt that the posh lift was not for humble mathematicians but for much more important people.

Thus started a friendship with the two Jans and with Edmund. I made several further visits to Poland and was able to see Warsaw be transformed into a modern city, not unlike any of the cities in Western Europe. Jan Krempa came back to Glasgow only once, as I recall, but Edmund made several visits. He would arrive in Glasgow en route from somewhere or other to Poland and always he had with him a sort of machete which he claimed

⁹Patrick Smith is a professor emeritus at Glasgow University. His email address is patrick.smith@talktalk.net

he always carried through security at airports! Twice I went with Edmund to his home in Białystok. He and his wife Maria could not have been kinder. The Białystok visits were particularly happy ones and gave opportunities to see rural Poland close up. It was always great fun spending time with Edmund. We certainly laughed a lot when we were together.

Edmund was a very good mathematician with a wide knowledge of the subject. Working with him was a great pleasure but very hard work. When he came to Glasgow he shared my office in the mathematics department which was not nearly as grand as the Palace but a good deal more comfortable! From the moment we both arrived at work on any given morning, we talked mathematics, almost always the particular problem we were working on at the time, bringing each other up to date with any progress made the previous evening. Most days, at some point I would have to go to give an undergraduate lecture. I would gather up my notes and head for the lecture hall and Edmund always came with me, still talking mathematics. When we got to the lecture hall he would reluctantly leave me to give my lecture and he returned to my office. When I got back to my office after the lecture, before I had even sat down, Edmund would start talking about mathematics, continuing the discussion we had had earlier. He was very stimulating to be with!

It was Edmund who introduced me to the work of his friend Kostia Beidar and with Kostia, Edmund and I wrote two articles about involution rings, one in 1994 and a sequel in 1997. Edmund had many interests in Algebra and mathematically we drifted apart after these papers while still remaining somewhat in touch and always delighted to meet each other at conferences. He enriched my life, mathematically and in many other ways, as I am sure he enriched the lives of many others.

Richard Wiegandt¹⁰

Edmund achieved substantial results in various branches of algebra. I shall concentrate to his and his students' contribution to radical theory and Köthe's Problem. Under the influence of Adam Suliński, from the mid-sixties my research interest turned to radical theory, and from the seventies also to his students' contribution. Edmund worked also in that direction. The common research interest with Edmund developed to cooperation, joint research, publications, and to close friendship.

Edmund was not a comet in the sky of Algebra, a unique phenomenon, but a talented, hardworking student of Adam Suliński's team, who became gradually an internationally recognized leading personality of Algebra and was appointed in 2000 as an editor of the leading mathematical journal *Communications in Algebra*. It is, therefore, necessary to go back to Suliński, present the creation of his team of algebraists and the development thereof.

I shall write in chronological order about Edmund's scientific career, mentioning our meetings worldwide, with nice, funny and strange experiences, not only with mathematicians. This recollection shows that the life of a research mathematician is not a boring routine, but a fascinatingly interesting one.

G. Köthe introduced the nil radical in 1930 as the maximal nil ideal of a ring which cannot be mapped onto a ring with nonzero idempotents. His aim was to represent rings as finite direct sums of matrix rings over division rings. This can be achieved by getting rid of the nil radical and by imposing descending chain condition on left ideals (i.e. artinian rings), resulting the Wedderburn-Artin Structure Theorem, a decomposition into a direct sum of simple rings. Thus rings with radical 0 have been called semisimple rings. Rad-

¹⁰Richard Wiegandt is a professor honoris causa at the National University of Mongolia. His email address is wiegandt@renyi.hu

ical theory has become the study of semisimple rings by considering also infinitely many components and various kinds of “simplicity” (e.g. rings of linear transformations) in the generalized versions of the Wedderburn-Artin Structure Theorem. Also R. Baer, N. Jacobson, J. Levitzki and N. H. McCoy with B. Brown introduced radicals, which provided the Wedderburn-Artin Structure Theorem for semisimple artinian rings, and the task remained to apply and refine G. Birkhoff’s Theorem stating that every algebraic structure, in particular every ring, is a subdirect product of subdirectly irreducible algebraic structures. Around 1953 S. A. Amitsur and A. G. Kurosh, independently, generalized and defined axiomatically the notions of radical and semisimplicity with the basic feature that there is a Galois correspondence between the class of radical rings and that of semisimple rings.

Spending a 6 months study leave in **Warsaw, 1964/65**, I got acquainted with Adam Suliński who obtained interesting results on Kurosh-Amitsur radicals jointly with Tim Anderson and N. J. (Tuzzie) Divinsky, in Vancouver. At that time, I worked on generalizing the Wedderburn-Artin Structure Theorem for linearly compact rings. Due to consultations with Adam I got inspirations to involve also Kurosh-Amitsur radicals in my investigations. During my **1969** visit to **Poland**, I conferred with Adam and spoke of continuing the category theoretical investigations of his 1966 paper.

In **June 1973** I spent two weeks in **Poland**. Mathematics at the Uniwersytet Warszawski was located in the Pałac Kultury i Nauki. I conferred with Adam Suliński and got acquainted with his team of algebraists, among them Michał Jaegerman, Jan Krempa, Edmund, Dorota Niewieczerał and Barbara Terlikowska. Michał dealt with Morita context in radical theory and Jan with Köthe’s Problem. In 1930 Köthe posed the problem: is every nil left ideal contained in the nil radical? Köthe’s Problem turned out

to be perhaps the most difficult and still unsolved problem of ring theory. A more interesting and attractive reformulation is due to Jan Krempa (1972): *are the polynomial rings over nil rings, Jacobson radical rings?* Edmund’s early publications were on other topics. Adam held the seminars as his Ph.D. supervisor A. G. Kurosh had done, sitting on the seat back in the last row, and directing the scientific discussion. Abode was provided in a private apartment around Żwirki i Wigury where among the playing children I saw a familiar looking girl. “Małgorzata, do you remember me?” — I asked her in English. In Islamabad we maintained friendly relation with the family of the Polish diplomat Andrzej Staszewski. Unexpectedly, I got an invitation for dinner by old acquaintances.



Figure 1: Adam Suliński

Warszawa. In **1978** the Banach Centrum organized the first Algebra Semester. It was located in a Pałacyk, Mokotowska 25, and I got accommodation in Mokotowska 57-59 in the same private apartment where I spent 6-month in 1964/65, a nice experience. I delivered a talk and conferred with Adam Suliński’s team at the University in the Pałac Kulturi I Nauki.



Figure 2: Edmund in Eger (1982)

The **1982** conference Radical Theory was held in **Eger, Hungary**, it was an encounter of algebraists from East and West, from all the five continents, and 14 excellent algebraists from the Soviet Union.

Entry visa was issued for Israeli algebraists, but not for South-Africans (though

the Proceedings published S. Veldsman's paper). N. Jacobson, S. A. Amitsur, N. Divinsky, A. D. Sands, Kostia Beidar, Efim Zelmanov and from Poland A. Suliński, Edmund, Jan Krempa, Bogdan Osłowski were among the 64 participants.



Figure 3: Edmund, Adam, Jacobson, Tuzzle

Jan talked about his famous example of a radical in a class of Lie algebras for which the ADS Property fails to hold, although its semisimple class is hereditary. Edmund's, Bogdan's and our joint paper with Adam was published in the Proceedings. The town Eger with its castle and many historical

buildings provided a pleasant environment for the Conference. The surrounding vineyards are famous for the red wine Bull's Blood which also contributed to the success of the Conference.

1983 Warszawa. By exchange program, I consulted with Adam Suliński's research team, including Edmund. Adam took me by car for sightseeing to Wilanów. That was the time of political unrest (*Solidarność*), and unthinkable shortage of goods even in food. Inviting for lunch at his home, Jan Krempa asked my preferred dish, *Leniwe* – a simple dish from *twaróg* (cottage cheese). I enjoyed the unbelievable rich and tasty dishes, but no *leniwe* was served, because there was no *twaróg* available.

In **Hobart, Australia 1987**, Barry Gardner organized a Conference on Rings, Modules and Radicals. Many leading experts participated in the Conference but not Edmund. In my talk I reported on Edmund's most recent results on general radical theory of nonassociative rings, obtained by applying category theoretical methods of Barbara Terlikowska-Osłowska. Surprisingly, Stefan Veldsman announced the same results gained indepen-

dently from Edmund. In a *universal class* (i.e. closed under taking ideals and homomorphic images) of nonassociative rings condition have been found to ensure affirmative answers to three major issues of general radical theory:

- The validity of the ADS-Theorem (Anderson, Divinsky, Suliński): the radical of any ideal of any ring is an ideal of the ring.
- The validity of Sands' Theorem: a class is semisimple if and only if it is regular, has the inductive property, and is closed under extensions.
- The Kurosh lower radical construction terminates at the first limit ordinal.

For a normal universal class, the three conditions are satisfied. Simultaneously, also Kostia Beidar answered the same question by using quite different methods and imposing adequate conditions. Kostia, actually continued the research of A. A. Nikitin.¹¹ A very special but deep problem was answered, simultaneously and independently by Edmund, Stefan, and Kostia, respectively – a strange phenomenon.

In February and March **1988** I took part of the Algebra Semester at the Banach Center, in Mokotowska, **Warszawa**. It was a good occasion to continue our scientific cooperation with Edmund, and also to get acquainted with Edmund's research collaborators Miguel Ferrero and Patrick Smith. Edmund lived in **Białystok**, and was a professor there, too. Upon his invitation I made a visit there, and conferred with his colleagues.

ICOR'88 was organized by Shoji Kyuno at Tohoku Gakuin University in **Sendai, Japan**. Edmund gave an account of joint results with Ewa Roszkowska on atoms in various lattices of radicals. The very generously organized conference culminated in a one-day

¹¹For details, see B. J. Gardner and R. Wiegandt: *Radical Theory of Rings*, Marcel Dekker 2004.

excursion to Matsushima, cruising by a boat to the cliff forest on the sea, spending the night in a brand new Japanese styled luxurious hotel, and having an authentic Japanese dinner with geishas.

In **1989** Edmund got a two-months invitation for joint research and lecturing under the auspices of the Hungarian research grant OTKA. During his stay he wrote two research papers, one of them jointly with Erzsébet Horváth on simple rings.



Figure 4: Ulaan Baatar (1990)

1990. Mongolia was, practically, the isolated backyard of the Soviet Union. The nomad inhabitants lived scattered in the vast territory, half a million in the capital Ulaan Baatar, partly in Soviet designed block houses, and partly in yurt settlements. 1990 was just the time of transition. The Mongol scientists earned C.Sc. and D.Sc. degrees in the Soviet Union and other socialist countries. The few, well trained Mongol algebraists organized a high-quality algebra conference with a dozen of foreign participants. Beside the nice, exotic city — and sight-seeing tours (tasting fermented mare milk, and Edmund turning the prayer mill), we experienced also some strange events. With Tomi and Jutka Schmidt we arrived on Sunday at noon to the empty hotel, and were stuck in the elevator for about 45 minutes. At the end of the conference a big bus took Edmund, the Schmidts and me with Mongol colleagues

to the airport, altogether only eight persons. The driver stood up, pushed the brake, and a hooligan through a big stone to the windshield. Tomi and Jutka sat just behind the driver, and all the glass shards fall on their head. The guy started to run towards the nearby yurt settlement. The driver left the road and chased the criminal by bus, entered the settlement and caught the hooligan. At the airport Tumurbat (earlier a wrestler, by now member of the Mongol Academy of Sciences) and the driver kept the criminal, and handed him over to the Police.



Figure 5: Edmund at the prayer mill

1991 Szekszárd, Adam and Edmund, attended ICOR'91 as well as Edmund's students Ryszard Andruszkiewicz, Piotr Grzeszczuk, Ryszard Mazurek, Ewa Roszkowska and Krempa's student Jan Okniński. Edmund's talk *Some questions concerning radicals of associative rings*, Mazurek's talk *The distributive radical* and Ewa's talk *Some result about modularity of lattices of radicals of associative rings* were published in the the Proceedings. Also Andruszkiewicz delivered a talk: *Accessible subrings of associative rings*, Grzeszczuk: *On constants of algebraic derivations*, and Okniński: *Prime and semiprime semigroup ring of cancellative semigroups*. In the dog-days the participants got acquainted with the historical town Szekszárd. In a conference dinner local wine sorts, in particular Bull's Blood of Szekszárd, were tasted.

Adam Suliński passed away in 1996, and on **ICOR'97**, in **Port Elizabeth**, Dorota Niewieczerzał spoke of Adam's contribution to radical theory.

With Edmund we met in **Brazil, July 1998**. Two algebra conferences were held, in **São Paulo** on Non Associative Algebras. Among the internationally known participant, quite a few immigrated from the collapsed Soviet Union. Jointly with his student Agata Smoktunowicz Edmund obtained an approximation of Köthe's Problem: *Polynomial rings over nil rings are Brown-McCoy radical rings*.



Figure 6: Brazil (1998)

São Paulo is located on the Tropic of Capricorn, 760 m above the sea level, with 22 million people, including also the surroundings. The traffic is chaotic, walking on the streets is not safe. The participants were accommodated in the business district, and the 10 km bus ride to the campus of the Universidade São Paulo took some 50 minutes Nevertheless, the huge campus is nice and interesting, in particular the pool of the venomous snakes. The food served for lunch, was excellent, the black bean dish, the food of slaves, tasted very well.

From São Paulo we flew to Porto Alegre, went by bus to **Canela**, attending the Escola de Algebra. Canela was founded by German settlers, and has become an attractive place for tourism. We enjoyed the phenomenal Brazil dinner Churrascaria, and visited



Figure 7: Canela (1998)

the nearby Parque do Caracol, and the waterfall Cascado do Caracol. Miguel Ferrero



Figure 8: Edmund, Miguel and Luisa

took us by car to his home in **Porto Alegre**. Miguel gave me accommodation for one night. Edmund remained there, working with Miguel on joint research. Although Porto Alegre is a safe city, the houses are equipped with security devices. Miguel opened the door from the car, after entering the car stopped, the door closed, and then the next door opened. Repeating this kind of precaution, we could reach Miguel's home.

ICOR '00 took place in the historical capital of Tyrol, **Innsbruck**, surrounded by beautiful mountains. The scientific highlights of the conference were linked with Köthe's Problem. Agata Smoktunowicz, as a keynote speaker, delivered a talk about her outstanding result: *Polynomial rings over nil rings*

need not be nil. She answered also Levitzki's Problem: *Simple nil rings do exist*, and so the nil and anti-simple radicals are not comparable. Agata came with her mother Alicja Smoktunowicz and uncle Ryszard Andruskiewicz both research mathematicians. Ryszard, whom I had known from ICOR'91, is an algebraist and a student of Edmund. The approximation the Köthe's Problem was improved jointly by Edmund, Kostia Beidar and Y. Fong, stating that *Polynomial rings over nil rings cannot be mapped homomorphically onto rings with nonzero idempotents*, i.e. the polynomial rings are a Behrens radical rings. As I met E. A. Behrens, and spoke of the important feature of the Berens radical, he got surprised "Ah, mein schiefgeburtetes Kind!". There was sightseeing in Inns-



Figure 9: Innsbruck (2000)

bruck, an ascent by cable car to the Hafelekar Spitze (2234 m), an excursion to South-Tyrol, Italy, through the Brenner Pass, offering a visit to Brixen/Bressanone, followed by a Tyrolian dinner in Neustift. An unexpected difficulty arose during the drive on the way to Brixen: on the old road, right under the Europabrücke, some 50 m ahead of the conference bus, a truck loaded with paper burned down, blocking the traffic for about 3 hours.

2001 Budapest. Maria accompanied Edmund for a visit to Budapest, and enjoyed the sightseeing and joint cultural programs.

ICM Satellite Conference Algebra and Re-



Figure 10: Bill Leavitt, Friederich Kasch



Figure 11: Edmund, Piri and Maria

lated Topics was held in **Hong Kong, 2002**. Edmund spoke of our joint result with Kostia Beidar, on an approximations of Köthe's Problem: *The polynomial rings over nil rings are uniformly strongly prime radical rings* (this radical had been introduced by D.M. Olson, 1987). The uniformly strongly prime radical is not comparable with the Behrens radical, but their intersection is still above the Jacobson radical.

Chişinău has a reputation of research in radical theory. **ICOR'03** was dedicated to the memory of A. V. Andrunakievich (1917-1997) founder of the Moldavian algebra school. Participants from all the five continents presented and discussed the most recent developments. In his talk Edmund pointed out that Köthe's Problem is equivalent to Andrunakievich's problem posed in 1969. There

was a full day excursion to the old fort Soroca, and one afternoon to the excavations of the town Orhei. At the end of the conference guests visited places where the famous Russian poet Pushkin spent his time in exile and got inspirations for his work.



Figure 12: Kazik Głazek

In **March 2005** the Conference of Algebra in Memory of Kostia Beidar was held in **Tainan, Taiwan**. Both Edmund and I did joint research with Kostia and were coauthors in 5 and 6 papers, respectively. In the Proceedings of the conference (Walter de Gruyter) we praised his contribution to radical

theory, detailed on 37 pages. Kostia wrote 30 substantial papers concerning radical theory, which was only a segment of his 161 publications, including 3 books. Kostia's reputation was indisputable; talking once on a surprising new result, someone asked who had proved it. "Beidar, who else?" — replied Arthur Sands. Among the internationally renowned participants was Kostia's contemporary, the Fields Medal awarded Russian-American Efim Zelmanov. All participants were gifted with a T-shirt with Kostia's portrait and "We love Beidar" printed on it.

On September 21, **2005** I got an e-mail from Kazimierz Głazek, inviting me to be a keynote speaker at the AAA Conference, coming February in Będlewo. He asked the title of my talk. I answered that next day I would fly to Ireland for sightseeing, and after returning I shall answer his request. Having returned from Ireland, I read the shockingly sad news that on September 26 Kazik had drowned in the hotel's pool in Tunis, where he had been on holiday with his wife. Our

friendship was a special one. Kazik was also a professional mountaineer in the Tatra, Alps and Himalayas (K2 and Nanga Parbat). He sent me documents, that half a century earlier also my parents had been mountaineers in the Tatra, moreover, also I was around the Nanga Parbat, but only at an elevation of 3000m, in summer, 1971.

As scheduled, the *Arbeitstagung Allgemeine Algebra* was held in **Będlewo** in February **2006**, in Memory of Kazimierz Głazek. The Potocki Palace and Park, Będlewo lies 30 km South of Poznań, and by German financial assistance, it has become a mathematical conference and research center, including also the Banach Center. At the AAA Conference Kazik was commemorated on a special session.



Figure 13: Białowieża (2006)



Figure 14: Bisons Żubry

After the AAA Conference I flew to

Warszawa, and Edmund immediately took me by train to **Białystok**. With Edmund we discussed some questions related to Köthe's Problem, and prepared the final version of our survey on Kostia's contribution to radical theory. Along with Marzena Filipowicz we took an excursion to **Białowieża**, on a bitterly cold winter day, and Edmund gifted me with Bison Grass for preparing the famous Polish hard drink Żubrówka. Back in Warsaw, I conferred with the algebraists. Edmund used to visit us yearly in November-December, and brought me supply to prepare Żubrówka.

ICOR'06 was organized by V.V. Kirichenko in the beautifully restored historical city, **Kyiv**. On the high quality conference Edmund met Halina France-Jackson, neé Korolczuk. Also Halina, professor in Port Elizabeth, was a student of Adam Suliński, and had close scientific contact with Edmund. At the conference dinner, the gourmet Rainer Mlitz asked dry wine, in Russian. The waitress was puzzled how can a liquid be dry.



Figure 15: Warszawa (2009)

ICOR'09 was held in **Warszawa**, at the Instytut Matematyczny PAN, Śniadeckich 8, and also the participants were accommodated there. Edmund organized the conference which was also referred to as a Workshop. Talks were given, among others, by Edmund, his student Marzena Filipowicz-Chomko, Agata Smoktunowicz, (professor in

Edinburgh), her student Michał Ziemkowski, Ryszard Andruszkiewicz and his student Karol Pryszczepko, Halina France-Jackson, Małgorzata Hryniewicka (student of Piotr Grzeszczuk), Izabela Malinowska and Jerzy Matczuk (both students of Jan Krempa), Ryszard Mazurek. Also P. Grzeszczuk, M. Kepczyk, J. Krempa, Dorota Niewieczerzał. J. Okniński, A. Strojnowski, Barbara Terlikowska-Osłowska attended the Workshop. After the Workshop I stayed two more days in Warszawa, toured the lovely places from Nowe Miasto to Wilanów and said farewell to the beloved city.



Figure 16: Marzena, Richard, Tumurbat, Halina

ICOR'12 was held in **Al Khoudh, Muscat Oman** at the luxurious Sultan Qaboos University, generously organized by Stefan Veldsman. The Afrikaner Stefan took up a professorship there in 1998. Edmund gave a talk *Some questions and new results on radicals of associative rings*, Halina spoke of **-rings in radical theory*, and Michał Ziemkowski *Some example of right McCoy ring*, beside other interesting talks. Due to the unrest in the Persian Gulf, the invited speaker Agata Smoktunowicz resigned.

The desert landscape is fantastic, Muscat is located on the coast, and not far from it rises the 3352 m mountain peak Jabal Shams. We visited the Jabal Shams Resthouse at the elevation 2700 m on a 24°C cold winter



Figure 17: Muscat (2012)



Figure 18: Muscat (2012)

day, admired the scenery around the 800 - 1000 m deep gaps. Settlements in the countryside receive water supply through a falaj (half a meter-wide open channel). From the 16th century for about 150 years the Portuguese conquerors built forts, which have become nowadays museums or cultural institution. The Omani Arabs are tall and slim, the cuisine resembles to the Indian and not to the Mediterranean one. The Omani Sultanate was an empire, the region from the Persian Gulf to Zanzibar was under its control. Spices from Zanzibar and all kind of special goods from India were brought by boat to Oman, and were carried, along with locally produced incense and dates, by camel caravans to the Mediterranean area. Slave market and slavery were banned in British colonies in 1835, and on French colonies in 1838. In 1970 (!) the reformer Sultan Qa-

boos, removing his father from power, banned slavery and slave market. Oman consists of 331 tribes. The Sultan rules as an absolute monarch. Beside the citizens, more than 50% of the population consists of temporary “guest workers”, in fact, relatively well paid slaves and servants, who can never settle and get citizenship. Sweeper, truck driver stewardess, mechanic, cook, travel agent, engineer, scientist, artist. There are no human rights, but Islamic sharia. Nevertheless, Sultan Qaboos was benevolent and liberal.



Figure 19: Arthur, Edmund, Heidi and Rainer



Figure 20: Stefan and Edmund

ICOR'15 was organized by Rainer Mlitz, and was held in **Vorau, Austria**, at the end of July. I hoped to spent again nice time with good friends, also with Edmund. Neither Edmund nor I could attend the conference, Edmund suffered a stroke in March, and I was

hospitalized for few days. After a long sickness Rainer passed away at the end of 2016. ICOR'15 was the last conference dedicated to radical theory.

The famous Köthe's Problem has been solved positively for various classes of rings, many equivalent formulations have been given, it has been approximated by radicals, but the problem posed in 1930 is still unsolved. Far too little is known about the possibility of transferring radical theoretical properties between a ring R and the polynomial ring $R[x]$. A considerable part of Edmund's research activity has been devoted to such kind of problems. It was Edmund who could have written a monograph about this topic. It is regretful that fate prevented him from doing so.

Drogi Edmundzie, spoczywaj w pokoju.

The punishment of long life is to write tributes to untimely passed away friends.