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Report on the PhD thesis of Łukasz Bożyk

The PhD dissertation of Łukasz Bożyk, *New results in (ob)structural graph theory*, made under the supervision of Michał Pilipczuk, presents several results in various aspects of structural and algorithmic graph theory. Before giving more details, let me first say that this is an outstanding work. To put it precisely, I have been in about twenty thesis committees, as a referee in about half of them, and I can say this is among the very best thesis that I had to read so far.

Structural graph theory is about graphs and their variants (oriented graphs, multi-graphs, ...), containment relations for them, obstructions that are particular graphs, and widths that are measures of complexity of graphs. The goal is to design efficient algorithms, taking some graph as an input, under some assumptions, typically that the graph has a small width or does not contain an obstruction (this sometimes allows a full description of the structure of the graphs under consideration). The complexity of the algorithm (that is its running time as a function of the graph) is often better viewed as parameterized, that is depending not only on the size of the input graph, but also on some parameter, often the width of the graph. All this topic broadly overlaps the topic of optimization, so dual notions are often the key to a deep understanding.

In all the aspects of structural graph theory explained above, Łukasz Bożyk presents interesting results, namely about *immersions* (a containment relation for graphs) for different types of graphs (tounaments and non-oriented graphs), the *Erdős-Pósa property* (that, when it is satisfied, provides a dual notion for containing obstructions), a new notion of width (the *ab-tree-cut width*) that works particularly well when dealing with immersions, several dual notions for this width (namely *brambles* and *tangles*), and a full structural description of a class of graphs, namely the *almost bipartite graphs*. Most of the contributions yield efficient algorithms for various problems.

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Contributions of the thesis

Chapter 1 is an introduction that very clearly presents what is structural graph theory and the main related concepts and challenges. It shows that the candidate is not only an expert of what he contributed to, but that he also has a broader knowledge. The rest of the manuscript is divided into three parts.

The first part is about immersions in tournaments. Immersion is a special containment relation, where edges of the contained graphs correspond to edge-disjoint paths of the container (and not vertex-disjoint paths as in the more famous notion of topological minor). Tournaments are oriented graphs arising by choosing one orientation for each edge of a complete graphs. Chapter 2 is a survey of known results on tournaments that are useful afterward. In Chapter 3, the Erdős-Pósa property is proved in tournaments for every oriented graph H, for both "immersion" and "topological minor" containment relation. This means that every tournament contains many disjoint copies of H, or a small set of vertices hits all copies of H. In Chapter 3, an algorithm is proposed to find such a small hitting set (for immersions of H). It is a kernilazation algorithm, so every instance is transformed into a smaller one, and this yields an FPT (fixed parameterized tractable) algorithm.

The second part of thesis is about notions dual to immersions. Tree-cut decomposition was originally invented by Wollan as a counterpart for immersions of the tree decompositions for minors. Bożyk describes a seemingly slight variant in the way to define a width out of it, the so-called ab-tree-cut width, that allows very nice developments. In particular, Chapter 5 contains in my opinion the nicest results of the thesis (though this is maybe a matter of taste): all the dual notions one can dream of when introducing a new notion of width are presented for the ab-tree-cut width: so-called "bramble", "tangle" and "cops-and-robbers" theorems. These are different dual notions to the tree-cut decompositions that mimic some key results of the Robertson and Seymour's Graph Minor Project. The idea of separating the measure of the width into adhesion-width and bag-width is very clever. The only missing thing is a variant of the celebrated grid theorem, but this was already

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discovered by Wollan. Chapter 6 explores possible variants of Wollan's theorem for digraphs. It is proved that a digraph that is sufficiently connected in some sense related to immersions (so based on the number of arc-disjoint paths linking some vertices) must contain (as an immersion) a digraph called the onion star. This is clearly a big step towards a wall or grid theorems for immersions in digraphs.

The third part of thesis is about the bipartite permutation deletion problem: the goal is to find an efficient algorithm for obtaining a bipartite permutation graph by deleting as few vertices as possible. A parameterized algorithm is presented for this problem (and one cannot hope more since the problem is NP-complete). An interesting feature of this algorithm is that it relies on a full structural description of a subclass, the so called almost bipartite permutation graphs. These full structural descriptions are always stricking, and like in the rest of the thesis, this is a very nice result.

Conclusion

To sum up, Łukasz Bożyk made very nice and original contributions to structural graph theory. Some of his results are of great interest regarding the theory, and others propose efficient algorithms. Many different techniques are used, all the proofs are very involved and technical, but still very clearly explained. Overall, the presentation excellent. All this shows a deep knowledge and a clear hability for research.

For all the reasons above, Łukasz Bożyk definitly deserves being awarded a PhD from the University of Warsaw. Moreover, I ask the University of Warsaw to consider this thesis for a honorary distinction.

Nicolas Trotignon, CNRS, LIP, ENS de Lyon.

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