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Jerzy Browkin (1934–2015)

by

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Jerzy Browkin was born on November 5, 1934 in Maciejów, Poland (now Ukraine). He studied mathematics at the University of Warsaw (1952–56) and then at the Lomonosov University in Moscow under the supervision of I. R. Shafarevich. He received a Ph.D. degree from the University of Warsaw in 1963 on submitting the thesis *Construction of the class field tower* (in Polish). Although formally his Ph.D. was supervised by A. Mostowski, he considered himself a student of W. Sierpiński and I. R. Shafarevich. Since 1955 he worked permanently at the University of Warsaw, where in 1969 he obtained his habilitation on submitting the thesis *Zeros of forms* (in Polish). In 1983 he was active in the organization of ICM in Warsaw. In 1987 he received the title of professor, which in Poland is given by the president of the state. During 1987–1991 he served as dean of the Faculty of Mathematics, Computer Science and Mechanics of the University of Warsaw. He retired from the University of Warsaw in February 2006 and from 2007 to 2015 he held a part-time position at the Institute of Mathematics of the Polish Academy of Sciences. In 1975–1991 he served as secretary of the Editorial Board of *Acta Arithmetica* and in 1991–2015 he was on the Advisory Board of the journal. He died on November 23, 2015.

Jerzy Browkin was the author or coauthor of 62 reviewed publications (the list is given below), 50 of which are research papers, 46 belonging to number theory. Here are Browkin's 15 coauthors arranged alphabetically: A. Białynicki-Birula, J. Brzeziński, H.-Q. Cao, D. Davies, B. Diviš, M. Filaseta, Ja. A. Gabovich, H. Gangl, G. Greaves, K. F. Hettling, B. Hoffmann, J. Hurrelbrink, A. Schinzel, E. Wirsing, K. Xu.

Browkin's scientific interests concerned elementary theory of numbers (mostly in the fifties), algebraic number theory (the sixties), zeros of forms

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in local fields (the seventies), arithmetic problems in algebraic K-theory (the eighties), the *abc*-conjecture (the nineties), computation of tame kernels of quadratic and cubic fields (1999–2004), elliptic curves since 2005. We shall discuss successively the results obtained by Browkin in each of these domains.

To elementary number theory (sections 11A, 11B, 11N of the MSC scheme) belong papers [2], [5], [6], [15], [16], [36], [42], [45], [47], [57], [62].

In [2] Browkin proves that the number of partitions $P(n)$ of a positive integer n satisfies $P(n+1) > P(n)$ for $n > 8$. This improves the result of Bateman and Erdős *[1]*, which states that $P(n+1) \geq P(n)$ for $n \geq 1$. In [15] the authors prove i.a. that if L is a field of characteristic p, $X \subset L$, $Y = \pm X$, $\operatorname{card} X = n$ and $p > 2^{n-1}$, then there exists $z \in L$ uniquely represented as $x + y$ with $x \in X$ and $y \in Y$. Let $f(p)$ be the maximal number such that in every set $\{a_1, \ldots, a_k\}$ of residues modulo p with $n \leq f(p)$ one of the differences $a_j - a_i$ occurs only once. Straus *[10]* proved that $f(p) \geq 1 + \frac{\log(p-1)}{\log 4}$. It follows from Browkin's result above that $f(p) \geq \frac{\log p}{\log 2}$.

In [16], [42] Browkin studies continued fractions for p-adic numbers. There are different definitions of a continued fraction expansion of an element of $\mathbb{Q}_p$. One such definition was given by Th. Schneider, another by Ruban *[9]*. Both definitions are quoted in [42]. Browkin's definition, which is a modification of Ruban's, was rediscovered by Wang *[12]*. It is the following: for $\alpha \in \mathbb{Q}_p$,

$$\alpha = b_0 + \frac{1|}{|b_1} + \frac{1|}{|b_2} + \cdots$$

where $b_j \in \mathbb{Z}[1/p] \cap (-p/2, p/2)$, $\operatorname{ord}_p b_0 = 0$, $\operatorname{ord}_p b_j < 0$ $(j < 0)$.

The following natural questions arise:

1. Can every $\alpha \in \mathbb{Q}_p$ be written uniquely as a finite or infinite continued fraction?
2. Can every rational number $\alpha \in \mathbb{Q}$ be written as a finite continued fraction?
3. Can every $\alpha \in \mathbb{Q}_p$ quadratic over $\mathbb{Q}$ be written as a periodic continued fraction?

For all definitions the answer to question 1 is yes; for Schneider's and Ruban's definition the answer to question 2 is no. For Browkin's definition the answer to the second question is yes, but the answer to the third question is unknown (see Bedocchi *[2]–[5]*).

In [36] the authors prove that there exist infinitely many integers not of the form $n - \varphi(n)$, however the existence of a sequence of such integers of positive density remains open (see Guy *[7]*, B36).

In [57] the authors prove that for any quadratic binomial $f(x) = rx^2 + s \in \mathbb{Z}[x]$ there exist positive integers $a \neq b$ such that $f(a), f(b)$ have the same prime factors and $\min\{a, b\}$ is arbitrarily large. The same is proved for a monic quadratic trinomial; for arbitrary quadratic trinomials the problem remains open.

To algebraic number theory, without K-theory, belong papers [8], [9], [58], [59] and [62]. In [8], [9] Browkin studies l-extensions such that only fixed primes $p_1, \ldots, p_n$ can ramify in them. In [58] and [59] the authors study finite groups exceptional in the following sense.

Let G be a finite group. Let

$$C_G(H) = \frac{1}{(G:H)} \sum_{H^* \text{ cyclic}, \, H \le H^* \le G} \mu(H^* : H),$$

where μ is the Möbius function. The group G is *exceptional* if $C_G(E) = 0$, where $E = \{1\}$ is the trivial subgroup of G.

The numbers $C_G(H)$ occur in the Brauer–Kuroda relation for the Dedekind zeta-function, which gives a connection with algebraic number theory.

In [62] Browkin gives examples of multiple zeros of the Dedekind zeta-function of the field $\mathbb{Q}(\zeta_3, \sqrt[3]{5})$.

To Diophantine equations (without the *abc*-conjecture) belong papers [1], [4], [7], [10], [12], [27], [50], [55], [56]; two of them, [10] and [12], concern zeros of forms in local fields. Browkin's attention was attracted by Terjanian's construction of a counterexample to Artin's conjecture that every form over $\mathbb{Q}_p$ of degree d with more than d^2 variables represents 0 non-trivially. Browkin proved that every form of "Terjanian type" with at least d^3 variables has a non-trivial zero in $\mathbb{Q}_p$. Later D. J. Lewis and H. L. Montgomery constructed for all primes p and for all $\varepsilon > 0$ infinitely many degrees d and forms of degree d over $\mathbb{Q}_p$ with at least $\exp(d/(\log d)(\log\log d)^{1+\varepsilon})$ variables without a non-trivial zero in $\mathbb{Q}_p$. In [27] Browkin gives the following theorem, nearly best possible.

Let p be a prime and $F_1, \ldots, F_k \in \mathbb{Z}[x_1, \ldots, x_n]$ be polynomials without constant term of degrees $d_1, \ldots, d_k$, respectively. Then the system of congruences

$$F_i(x_1, \ldots, x_n) \equiv 0 \bmod p^{m_i}, \quad i = 1, \ldots, k,$$

has a solution $a_1, \ldots, a_n \in \mathbb{Z}^n$ with not all a_j's divisible by p provided

$$n > \sum_{i=1}^{k} d_i \, \frac{p^{m_i} - 1}{p - 1}\,.$$

In [50] the authors propose the following conjecture.

Let $x_1,\dots,x_n$, $n>2$, be positive integers satisfying

$$x_{i-1}^2-2x_i^2+x_{i+1}^2=\Delta \quad \text{for } i=2,\dots,n-1.$$

(a) *If $\Delta=2$ and $n>4$, then $x_1,\dots,x_n$ are consecutive integers.*
(b) *If $\Delta\neq 2$ and $n>8$, then $\Delta=2\delta^2$ and there is an arithmetic progression $y_1,\dots,y_n$ with difference δ such that $x_i=|y_i|$.*

To arithmetic problems in algebraic K-theory Browkin devoted two survey papers [22], [33] and research papers [23], [26], [28], [29], [32], [34], [38], [40], [43], [47], [51], [54], [60], the first of which has been the most often cited of Browkin's papers.

Let $F=\mathbb{Q}(\sqrt{d})$, where d is a square-free integer, and let e_n be the number of cyclic factors of K_2O_F whose order is divisible by n. It is shown in [23] that if $d>2$, then $e_2=s+t$, where t is the number of odd primes dividing d and 2^s is the number of elements of $\{\pm1,\pm2\}$ that are norms from F.

In [34] Browkin proves some conjectures of H. Gangl based on numerical computation.

In [38] the authors give, for all quadratic imaginary number fields of discriminant $d>-5000$, the conjectured value of the order of K_2O_F. The conjecture agrees with the few orders determined rigorously.

In [47] Browkin gives the results of computation of the structure of K_2O_F for all cyclic cubic fields with only one ramified p, $7\le p<5000$.

To the *abc*-conjecture Browkin devoted a survey paper [41] and research papers [35], [37], [39], [49], [52]. In [35] (joint with J. Brzeziński) there are 11 examples not found earlier of equalities

$$a+b=c,\quad a,b \text{ coprime positive integers},$$

with

$$L=\frac{\log c}{\log r(abc)}>1.4,\quad r(abc) \text{ the radical of } abc,$$

among them the equality

$$19\cdot1307+7\cdot29^2\cdot31^8=2^3\cdot3^{22}\cdot5^4,$$

where $L\approx1.623490$, which is the third known largest L. Moreover, in [49] Browkin proposed the form of the *abc*-conjecture for all algebraic numbers, different from the forms proposed by Broberg *[6]* and Masser *[8]*.

To elliptic curves Browkin devoted a survey paper [44] and one research paper [53] with D. Davies about Kodaira classes of elliptic curves over $\mathbb{Q}$. It is known that for a prime $p>3$ quadratic twists permute the Kodaira classes. The authors of [53] establish a refinement of the Kodaira classification that ensures that the permutation property is recovered by refined classes for $p=2$ or 3.

Finally, in paper [11] on the border of number theory and set theory, the authors give a negative answer to a problem of Ulam *[11]*.

Reviewed publications of Jerzy Browkin

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